Appendix: Social welfare maximization under two perspectives

1. The central planner perspective

In a centrally planned economy, a central planner is responsible for the production and the distribution of a particular good. The aim is to maximise the aggregate social welfare by identifying the optimal quantity to be produced by each firm and the optimal quantity to be consumed by each household. What are the necessary conditions to maximize social welfare? The formal mathematical treatment is given below.

Assumptions:

1. There are N_p firms (producers), indexed by *i* or *k*, $i(k)=1,\ldots,N_p$
2. There are N_c households (consumers), indexed by *j* or *l* $(j(l)=1,\ldots., N_c$
3. The total cost of producer *i* is noted $TC_i(q_i^S)$, where q_i^S is the produced quantity (*S* for supply), and the marginal cost, $MC_i = \partial TC_i / \partial q_i^S$, is increasing ($\partial MC_i / \partial q_i^S$ >0)
4. The total benefit of consumer *j* is noted $TB_j(q_j^D)$ where q_j^D is the consumed quantity (*D* for demand), and the marginal benefit, $MB_j = \partial TB_j / \partial q_j^D$, is decreasing ($\partial MB_j / \partial q_j^D$ <0)
5. The good is of homogeneous quality and perfectly divisible, and all benefits and costs are summable i.e. one monetary unit less for any one agent is perfectly compensated by one monetary unit more for any other agent
6. The total produced quantity is equal to the total consumed quantity ($\sum_{i=1}^{N_p} q_i^S = \sum_{j=1}^{N_c} q_j^D$)
7. All costs and benefits are perfectly known by the central planner
8. The central planner can impose to each producer the quantity to produce, and to each household the quantity to consume

Mathematically, the problem is solved by maximising the following Lagrangian:

$$\max_{q_i^S, q_j^D, \lambda} L = \sum_{j=1}^{N_c} TB_j(q_j^D) - \sum_{i=1}^{N_p} TC_i(q_i^S) + \lambda\left(\sum_{i=1}^{N_p} q_i^S - \sum_{j=1}^{N_c} q_j^D\right)$$

The first-order conditions (FOC) are given by :

$$\frac{\partial L}{\partial q_i^S} = 0 \Leftrightarrow MC_i(q_i^{S*}) = \lambda \quad \forall i = 1,\ldots, N_p \qquad [1]$$

$$\frac{\partial L}{\partial q_j^D} = 0 \Leftrightarrow MB_j(q_j^{D*}) = \lambda \quad \forall j = 1,\ldots, N_c \qquad [2]$$

$$\frac{\partial L}{\partial \lambda} = 0 \Leftrightarrow \sum_{i=1}^{N_p} q_i^{S*} = \sum_{j=1}^{N_c} q_j^{D*} = q^* \qquad [3]$$

Where the quantities produced and consumed are marked by an asterisk (*) to indicate that these are the optimal quantities (maximising the net social welfare).

Relations [1] correspond to the equalisation of marginal costs between all producers, or « cost-effectiveness » condition. It answers the question: « how to distribute q* among producers so that the sum of total production costs is minimized? ».

Relations [2] correspond to the equalisation of marginal benefits between households, or « benefit-effectiveness » condition. It answers the question: « how to distribute q* among the households so that the sum of total consumption benefits is maximized? ».

Relation [3] corresponds to the equalization of total supply and total demand.

The combination of [1] and [2] implies that marginal cost (unique at the optimum) is equal to marginal benefit (unique at the optimum).

A huge task. To achieve the stated objective, the central planner has to solve a system of N_p+N_c+1 unknown variables, taking into account the specificities of the production function of each firm and of the benefit function of each household. Moreover, to implement the optimal outcome at the level of each agent, the central planner needs a huge and highly efficient administration to impose (without cost nor cheating) the solution to each actor. This is the meaning of assumptions [6], [7] and [8].

2. The market perspective

Nobody is in charge. Imagine now that the same production-consumption problem but in the context of a decentralized market economy. The quantities produced and consumed are the result of a myriad decentralised decisions. Each producer maximises her or his own profit. Each household maximises her or his satisfaction. Each economic agent is perfectly selfish, independent and totally disregards the result of her or his choice for the rest of society.

Assumptions. Assumptions 1-5 are unchanged. Assumptions 6-8 are replaced by :

vi. each agent knows perfectly the specific cost conditions (if it is a producer) or benefit conditions (if it is a household), as well as the price (and the quality) of the good
vii. each agent is atomistic (or "price taker"), which means too small to influence the market price and considers the price as given
viii. There are no externalities, neither on the production nor consumption side, all the costs are borne by the individual producer, all the benefits are gained by the individual consumer, with no benefit nor cost on third parties.

The invisible hand diagram. As shown on the left of figure, each producer equalizes the price with the marginal cost, which leads to individual supplies and then to the market supply. On the right, each household equalizes the price with the marginal benefit, which leads to individual demands and to market demand. The intersection between the supply and the demand curves leads to p*, q*, which is the only price-quantity combination so that total supply is equal to total demand. When this equilibrium is reached, the FOC efficiency conditions [1], [2] et [3] are automatically satisfied without anyone intervening, as if an « invisible hand » had guided the individual decisions in the good direction.

Caveats. The result crucially depends on assumptions vi to viii. The main merit of Adam Smith's parable is to illustrate the potential advantages of a system of decentralised decisions, *when the conditions are fulfilled* so that the price variations replace, harmoniously (i.e. at a much lower cost), the planners' decisions.

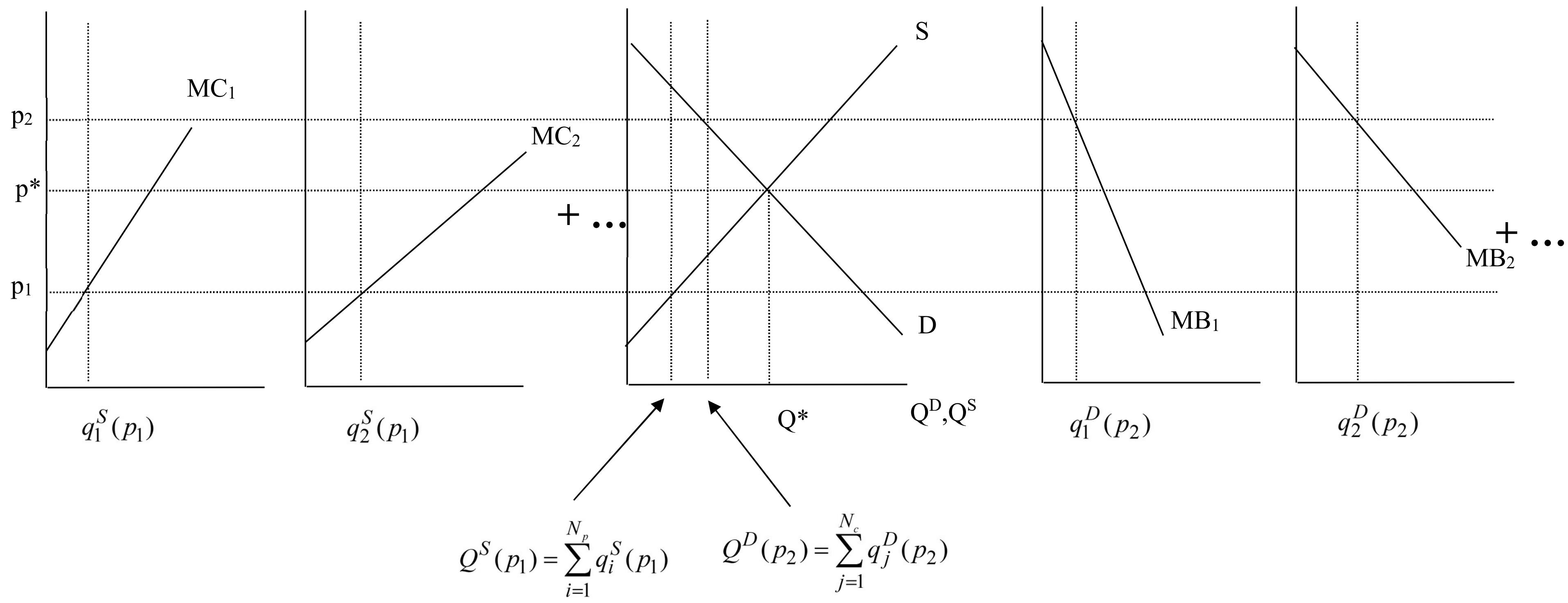

Figure A1 : Demand, supply and market equilibrium