Principles of sustainability economics: Extended correction guide

Chapter 6, problems 6.1 to 6.6

Problem 6.1: Quantity restriction

i. For sector 1, as emission and production units are identical, we can simply replace Q_1 by E_1 in all marginal expressions for Q_1, which leads to $MNB_1^E = 10 - E_1$. For sector 2, it is less immediate. Following the hint, we get first the expressions for total benefits and total costs by integrating the functions of MB_2^Q and MC_2^Q (or calculating the area below the corresponding schedules, see Technical Appendix A1) to obtain: $TB_2 = 10Q_2 - 0.25(Q_2)^2$ and $TC_2 = 0.25(Q_2)^2$. Then, we replace Q_2 by $0.5E_2$ and get: $TB_2 = 5E_2 - 0.0625(E_2)^2$ and $TC_2 = 0.0625(E_2)^2$. Finally, taking the derivative with respect to E_2 we obtain $MB_2^E = 5 - 0.125E_2$ and $MC_2^E = 0.125E_2$, which implies that $MNB_2^E = 5 - 0.25E_2$.

Note. One could be tempted to say that the above method followed for sector 2 is unnecessarily complicated, and suggest that directly substituting $0.5E_2$ for Q_2 into the marginal expressions in Q_2 would be quicker. But in this case we would obtain $MB_2^E = 10 - 0.25E_2$ and $MC_2^E = 0.25E_2$, which would be both **wrong**. Where does the mistake come from? Basically, because when we substitute directly we fail considering the fact that the derivative variable itself has changed. Generally speaking, the marginal benefit with respect to emission units should be written $MB_2^E = \partial TB_2/\partial E_2$ where TB_2 is a function of Q_2 *which is itself a function* of E_2. The same reasoning applies $MC_2^E = \partial TC_2/\partial E_2$. By applying the chain derivation rule we obtain: $MB_2^E = \partial TB_2/\partial E_2 = [\partial TB_2/\partial Q_2][\partial Q_2/\partial E_2] = MB_2^Q[1/2]$, $MC_2^E = \partial TC_2/\partial E_2 = [\partial TC_2/\partial Q_2][\partial Q_2/\partial E_2] = MC_2^Q[1/2]$. Substituting $0.5E_2$ for Q_2 in MB_2^Q and MC_2^Q we obtain the correct expressions.

ii. The diagrams below are from the excel file *problem_6.1.xlsx*. The inverted expressions of the marginal net benefit curves $MNB_1 = 10 - E_1$ and $MNB_2 = 5 - 0.25E_2$ are given by $E_1 = 10 - p$ and $E_2 = 20 - 4p$ where p represents the variable on the vertical axis. If $p > 5$, only sector 1 is considered. If $p \leq 5$ we add up the two sources of emissions to obtain: $E = E_1 + E_2 = 30 - 5p$, or $MNB^E = 6 - 0.2E$, which corresponds to the lower part of the kinked curve on the right panel.

See the completed Excel file *problem_6.1.xlsx*

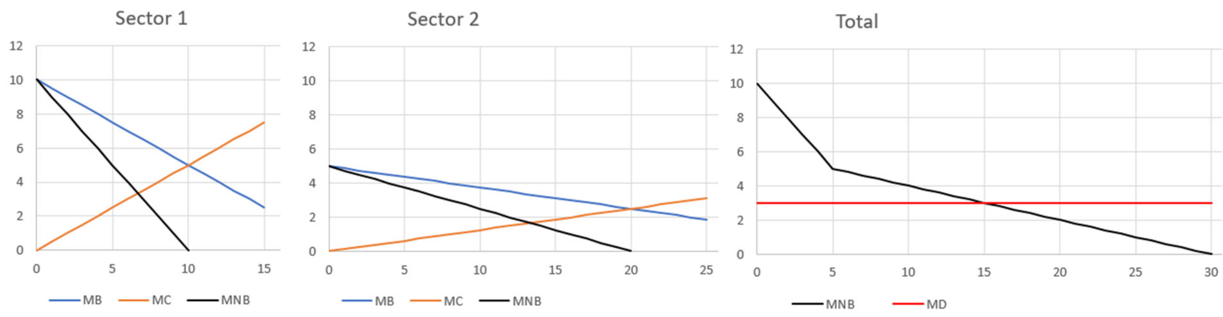

Figure C6.1: Socially efficient level of emissions

At the market equilibrium the net marginal benefit is equal to 0 in each sector. This leads to $E_1 = 10$ and $E_2 = 20$. Note that this is equivalent to equalizing marginal benefit with marginal cost: $MB_1^E = MC_1^E \Rightarrow 10 - 0.5E_1 = 0.5E_1 \Rightarrow E_1 = 10$ and $MB_2^E = MC_2^E \Rightarrow 5 - 0.125E_2 = 0.125E_2 \Rightarrow E_2 = 20$.

The social optimum is obtained at the intersection between the MNB^E curve and the marginal damage (MD) curve i.e. $6 - 02E = 3 \Rightarrow 0.2E = 3 \Rightarrow E = 15$ and $p = MNB^E = MD = 3$. This leads to $E_1 = 10 - 3 = 7$ and $E_2 = 20 - 4(3) = 8$. Logically, a major reduction of emissions is imposed on the dirtiest sector.

The change in social welfare is given by the difference between the reduction in damage costs (area between 15 and 30 below the MD curve) and the reduction in net private welfare (area below the MNB^E curve). Numerically this gives: $[15(3)] - \left[15\left(\frac{3}{2}\right)\right] = \left[15\left(\frac{3}{2}\right)\right] = 22.5$.

iii. The aggregate reduction in emissions is 15 units, thus each sector produces $15/2 = 7.5$ units. In comparison with the social optimum situation, as illustrated by figure C6.2, this leads to a change in net benefits of $+acde$ for sector 1 and of $-fghi$ for sector 2. The net change in social welfare is therefore given by $-(fgj + abc) = -(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}) = -\frac{5}{32}$. The principle of TBM is violated, because both sectors do not produce the quantity at the intersection between the MNB_i curve and the MD curve (as determined in point ii).

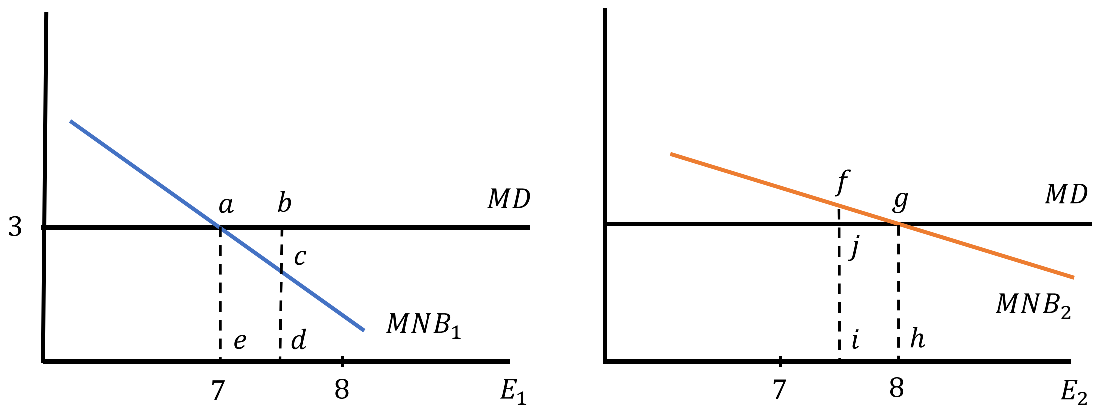

Figure C6.2: Identical reduction of emissions in each sector

iv. If the MD curve increases, the quantity produced by each sector at the optimum decreases. This means that quantity restrictions increase as the MD curve shifts up. The critical thresholds are achieved when the MD curve is greater than the MNB curve for each sector. Beyond these thresholds, the quantity produced becomes zero. Sector 2 no longer produces when $MD = 5$ and sector 1 stops producing when $MD = 10$.

Problem 6.2: Abatement

i. Let us call $MAC = \Delta$, from the MAC_i expressions we get $A_1 = 9\sqrt{\Delta}$ and $A_2 = 3\sqrt{\Delta}$ which leads to $A = A_1 + A_2 = 12\sqrt{\Delta} \Rightarrow MAC = \left(\frac{A}{12}\right)^2$. The optimum is achieved when $MAC = MD = 3 \Rightarrow \left(\frac{A}{12}\right)^2 = 3 \Rightarrow A = 12\sqrt{3}$, $A_1 = 9\sqrt{3}$ and $A_2 = 3\sqrt{3}$.

See the completed Excel file *problem_6.2.xlsx*

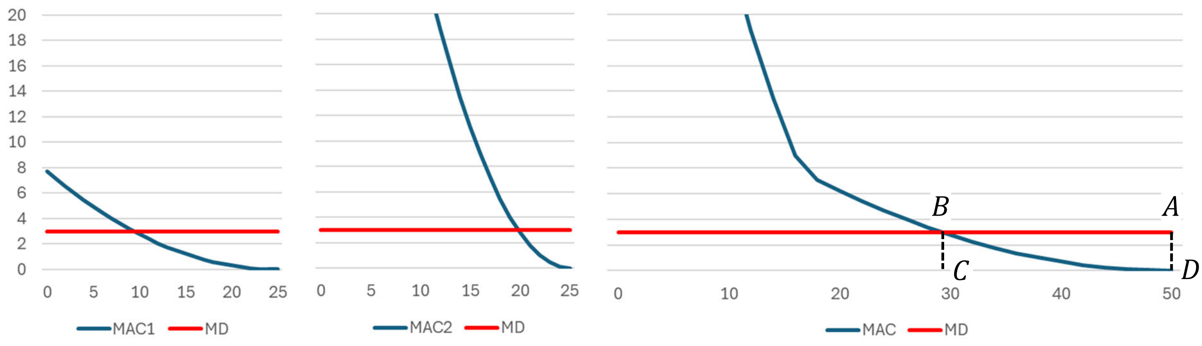

Figure C6.3: Socially efficient abatement – two abatement technologies

To plot the curves, be aware that emission levels are on the horizontal axis, so abatement levels should be considered from right to left on the same axis. Knowing that $E^p = 50$ and $E = E^p - A$, we can set $MAC = \left(\frac{50-E}{12}\right)^2$. Following the same approach and knowing that $E_1^p = E_2^p = \frac{E^p}{2} = 25$, we can set $MAC_1 = \left(\frac{25-E_1}{9}\right)^2$ and $MAC_2 = \left(\frac{25-E_2}{3}\right)^2$ (with the restriction that $0 \le E_1, E_2 \le 25$). The aggregated MAC curve shows a kink due to the difference between the two technologies, occurring when abated emissions from technology 1 reach their maximum ($E_1 = 0$):

$$MAC = \begin{cases} \left(\frac{A-25}{3}\right)^2 \ if\ MAC > \left(\frac{25-0}{9}\right)^2 \cong 7.71 \\ \left(\frac{A}{12}\right)^2 \ if\ MAC \le \left(\frac{25-0}{9}\right)^2 \cong 7.71 \end{cases}.$$

To determine the net welfare gain, we consider the social benefit resulting from the reduction of damages given by area $ABCD = 3A = 3(12\sqrt{3}) = 36\sqrt{3}$ and the abatement costs given by area $BCD = \int_0^{12\sqrt{3}} \left(\frac{A}{12}\right)^2 dA = \left[\frac{1}{3}\left(\frac{A}{12}\right)^3 \cdot 12\right]_0^{12\sqrt{3}} = \left[\frac{1}{3 \cdot 12^2} A^3\right]_0^{12\sqrt{3}} = 12\sqrt{3}$. Therefore, the net welfare gain is given by $ABCD - BCD = 24\sqrt{3}$.

ii. If both technologies abate the same level, we have $A_1 = A_2 = \frac{A}{2} = 6\sqrt{3}$. We can see from the following figure that this leads to a net change in benefits of $+acde$ for technology 1 and of $-fghi$ for technology 2. The net change in welfare is therefore given by $-(abc + fgj)$.

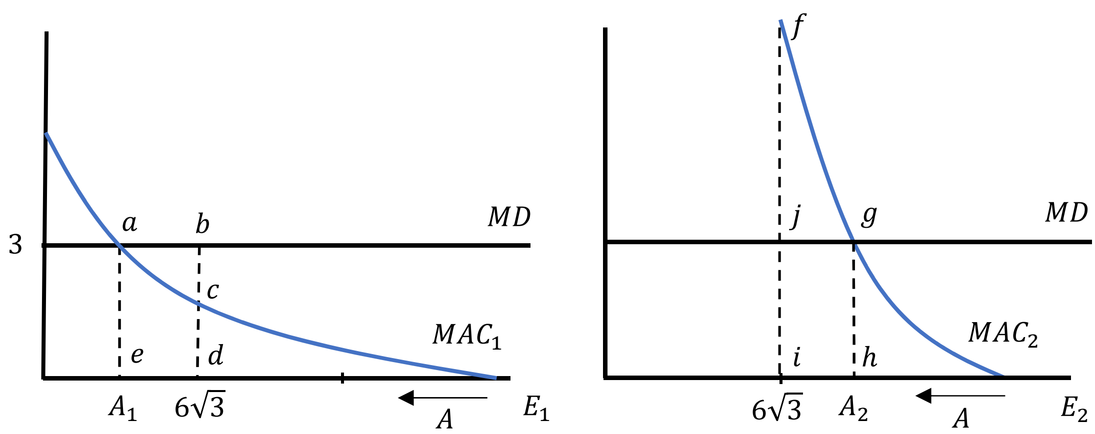

Figure C6.4: Identical reduction of emissions in each sector

The area $abc = abde - acde$ where $abde = 3(9\sqrt{3} - 6\sqrt{3}) = 9\sqrt{3}$ and $acde = \int_{6\sqrt{3}}^{9\sqrt{3}} \left(\frac{A_1}{9}\right)^2 dA = \left|\frac{1}{3}\left(\frac{A_1}{9}\right)^3 \cdot 9\right|_{6\sqrt{3}}^{9\sqrt{3}} = \left|\frac{1}{3\cdot 9^2} A_1{}^3\right|_{6\sqrt{3}}^{9\sqrt{3}} = 9\sqrt{3} - \frac{8}{9}\sqrt{3^3} = 9\sqrt{3} - \frac{8}{3}\sqrt{3} = \frac{19}{3}\sqrt{3}$. We thus obtain $abc = 9\sqrt{3} - \frac{19}{3}\sqrt{3} = \frac{8}{3}\sqrt{3}$. Similarly, the area $fgj = fghi - jghi$ where $fghi = \int_{3\sqrt{3}}^{6\sqrt{3}} \left(\frac{A_2}{3}\right)^2 dA = \left|\frac{1}{3}\left(\frac{A_2}{3}\right)^3 \cdot 3\right|_{3\sqrt{3}}^{6\sqrt{3}} = 8\sqrt{3^3} - \sqrt{3^3} = 24\sqrt{3} - 3\sqrt{3} = 21\sqrt{3}$ and $jghi = 3(6\sqrt{3} - 3\sqrt{3}) = 9\sqrt{3}$. We thus obtain $fgj = 21\sqrt{3} - 9\sqrt{3} = 12\sqrt{3}$. Therefore, the net change in welfare is $-(abc + fgj) = -(\frac{8}{3}\sqrt{3} + 12\sqrt{3}) = -\frac{44}{3}\sqrt{3}$.

The principle of TCM is violated, because both sectors do not abate the quantity at the intersection between the MAC_i curve and the MD curve (as determined in point i). Technology 1 is cheaper, so it should abate more than technology 2.

iii. The intuition is similar to point iv. of problem 6.1. If the MD curve increases, the abatement effort by each technology at the optimum increases. The critical thresholds are achieved when the MD curve reaches the maximal abatement effort i.e. when $A_1 = A_2 = 25$. Beyond these thresholds, if the MD curve continues to rise, the abatement effort does not increase. To determine the thresholds, we set $A_1 = 25 \Rightarrow MD = \left(\frac{25}{9}\right)^2 \cong 7.72$ and $A_2 = 25 \Rightarrow MD = \left(\frac{25}{3}\right)^2 \cong 69.44$. The higher value is explained by the higher abatement cost of technology 2 compared with technology 1.

Problem 6.3: Optimal combination of restriction and abatement

i. To draw the graph of figure 6.6, we need to determine the expression for social marginal cost SMC. The expression [3] in section 6.1.4 is given by $SMC(Q) = \frac{dSTC}{dQ} = MD\sqrt{\frac{Q_0}{Q}}$. We know that $p_a = MD = 3 \Rightarrow Q_0 = 1$, so we get $SMC = MD\frac{1}{\sqrt{Q}} = \frac{MD}{\sqrt{Q}} = \frac{3}{\sqrt{Q}}$. At the optimum, the social efficiency condition is achieved when $SMC = MNB^Q \Rightarrow \frac{3}{\sqrt{Q}} = \frac{20}{3} - \frac{4}{3}\sqrt{Q}$. We then define $q = \sqrt{Q}$ and replace it in the above expression to obtain $q(20 - 4q) = 9 \Rightarrow 4q^2 - 20q + 9 = 0$. Using the formula for solving a quadratic function, we get $\Delta = \sqrt{400 - 144} = 16 \Rightarrow q_1 = \frac{20-16}{8} = \frac{1}{2}, q_2 = \frac{20+16}{8} = \frac{9}{2} \Rightarrow Q_1 = \frac{1}{4}, Q_2 = \frac{81}{4}$.

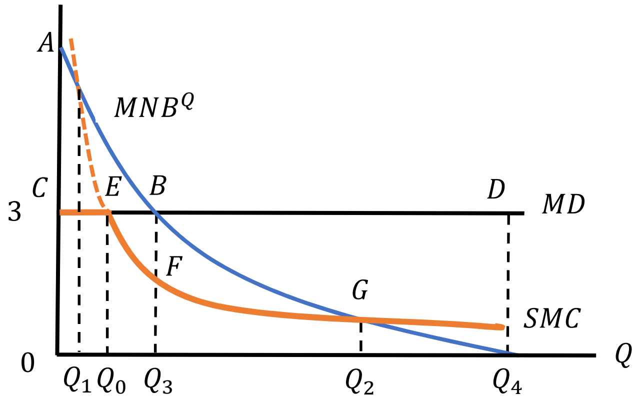

Figure C6.5: The social marginal cost

Remember from section 6.1.4 that Q_0 represents the critical quantity level, where the optimal input mix for Q, i.e. $\frac{MD}{p_a} = \frac{1+a}{E}$ implies the absence of abatement activities ($a = 0$). In other words, this is the threshold below which abatement becomes pointless because it would have to be negative to achieve the optimal combination of inputs. Therefore, the SMC curve (solid orange line) presents a kink at $Q = Q_0$ and the quantity $Q_1 = \frac{1}{4} < Q_0$ is not an optimal quantity at the social optimum.

ii. In order to calculate net social welfare, we first need to determine the quantity produced in the three different situations. The socially optimal quantity when quantity restriction as well as abatement activities are possible was determined at point i, i.e. $Q_2 = \frac{81}{4}$. The social optimum when considering quantity restriction only is given by the quantity such that $MNB^Q = MD \Rightarrow \frac{20}{3} - \frac{4}{3}\sqrt{Q} = 3 \Rightarrow \frac{4}{3}\sqrt{Q} = \frac{11}{3} \Rightarrow \sqrt{Q} = \frac{11}{4} \Rightarrow Q_3 = \frac{121}{16} = 7.5625$. At the market equilibrium, when neither pollution damages nor abatement activities are considered, the quantity produced is given by the point where $MNB^Q = 0 \Rightarrow \frac{20}{3} - \frac{4}{3}\sqrt{Q} = 0 \Rightarrow Q_4 = 25$.

The net social welfare at the market equilibrium (Q_4) is given by subtracting the area below the MD curve from the area below the MNB^Q curve from $Q = 0$ to Q_4. We get:

$$AQ_4O - CDQ_4O = ABC - BDQ_4 = \int_0^{Q_3}(MNB^Q - MD)\,dQ - \int_{Q_3}^{Q_4}(MNB^Q - MD)\,dQ = \int_0^{Q_3}\left(\frac{11}{3} - \frac{4}{3}Q^{\frac{1}{2}}\right)dQ - \int_{Q_3}^{Q_4}\left(\frac{4}{3}Q^{\frac{1}{2}} - \frac{11}{3}\right)dQ = \left[\frac{11}{3}Q - \frac{8}{9}Q^{\frac{3}{2}}\right]_0^{Q_3} - \left[\frac{8}{9}Q^{\frac{3}{2}} - \frac{11}{3}Q\right]_{Q_3}^{Q_4} = \left[\frac{1331}{48} - \frac{8}{9}\left(\frac{1331}{64}\right)\right] - \left[\left(\frac{8}{9}(125) - \frac{275}{3}\right) - \left(\frac{8}{9}\left(\frac{1331}{64}\right) - \frac{1331}{48}\right)\right] = \left(\frac{1331}{144}\right) - \left(\frac{2800}{144} - \left(-\frac{1331}{144}\right)\right) = -\frac{2800}{144} = -19.\overline{4}.$$

The net social welfare at the restriction-only equilibrium (Q_3) is given by subtracting the area below the MD curve from the area below the MNB^Q curve from $Q = 0$ to Q_3. We get:

$$ABQ_30 - CBQ_30 = ABC = \int_0^{Q_3}(MNB^Q - MD)\,dQ = \frac{1331}{144} \cong 9.24.$$

The net social welfare at the restriction-cum-abatement optimum (Q_2) is given by subtracting the area below the SMC curve from the area below the MNB^Q curve from $Q = 0$ to Q_2. The social gain is given by $AGQ_20 - CEGQ_20 = ABC + EBF + BGF$, where ABC is the part of gain similar to the two previous optimality situations and $EBF + BGF$ corresponds to the improved cost-efficiency due to optimal mixing with abatement activities. We obtain:

$EBF = \int_{Q_0}^{Q_3}(MD - SMC)\,dQ = \int_{Q_0}^{Q_3}\left(3 - 3Q^{-\frac{1}{2}}\right)dQ = \left[3Q - 6Q^{\frac{1}{2}}\right]_{Q_0}^{Q_3} = \left[\left(\frac{363}{16} - \frac{33}{2}\right) - (3 - 6)\right] = \frac{147}{16} = 9.1875$ and $BGF = \int_{Q_3}^{Q_2}(MNB^Q - SMC)\,dQ = \int_{Q_3}^{Q_2}\left(\frac{20}{3} - \frac{4}{3}Q^{\frac{1}{2}} - 3Q^{-\frac{1}{2}}\right)dQ = \left[\frac{20}{3}Q - \frac{8}{9}Q^{\frac{3}{2}} - 6Q^{\frac{1}{2}}\right]_{Q_3}^{Q_2} = (135 - 81 - 27) - \left(\frac{605}{12} - \frac{1331}{72} - \frac{33}{2}\right) = \left(27 - \frac{1111}{72}\right) \cong 11.57.$

Therefore, the social gain due to optimal mixing is $EBF + BGF \cong 20.76$ and the total social gain is $ABC + EBF + BGF \cong 20.76 + 9.24 = 30$.

iii. The quantity produced at the social optimum is $Q_2 = \frac{81}{4}$.

We can determine the isoquant function by using the expressions [2] in section 6.1.4, which give the optimal inputs as a function of production level. By plugging known values into these expressions, we obtain $E^* = E(Q_2) = \sqrt{Q_2 Q_0} = \sqrt{20.25 \cdot 1} = \sqrt{\frac{81}{4}} = 4.5$ and $a^* = a(Q_2) = \sqrt{\frac{Q_2}{Q_0}} - 1 = \sqrt{\frac{81}{4}} - 1 = 3.5$. The isoquant representing the optimal input mix, i.e. the optimal combination of E and a for a fix quantity Q_2, is given by $Q_2 = E(a+1) \Rightarrow a + 1 = \frac{Q_2}{E} \Rightarrow a = Q_2 \cdot \frac{1}{E} - 1 \Rightarrow a = \frac{81}{4} \cdot \frac{1}{E} - 1$. As shown in figure C6.6, Q_2 is located at the point where the isoquant crosses the horizontal axis ($a = 0$). Note that the slope of the isoquant at the social optimum is equal to $\frac{da}{dE} = -Q_2 \cdot \frac{1}{E^2} = -\frac{81}{4}\frac{1}{\left(\frac{9}{2}\right)^2} = -1$.

The iso-cost curve represents the (E, a) combinations such that social total cost remains constant. The value of $\overline{STC}$ can be determined by plugging the optimal values of E^* and a^* into the expression $\overline{STC} = p_a \cdot a + MD \cdot E = 3(3.5 + 4.5) = 24$. Taking the initial expression and rearranging in terms of a, we obtain $24 = 3(a + E) \Rightarrow a = 8 - E$ i.e. a straight line with a slope of -1, confirming the tangency between the isoquant and the iso-cost curve.

Recall that abated emissions are obtained by subtracting the level of emissions from the quantity produced, such that $A^* = Q_2 - E^* = \frac{81}{4} - \frac{9}{2} = \frac{63}{4} = 15.75$.

The above expressions can be graphically represented as follows:

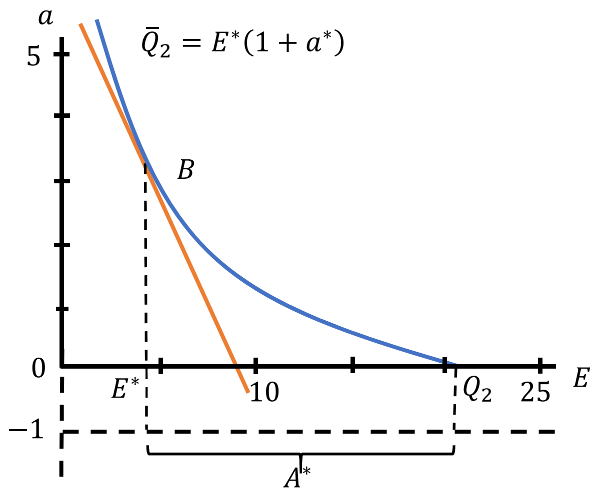

Figure C6.6: The optimal mix

iv. Let's use the usual notation of growth rates $\hat{x}$ to define the percentage change of variable x, i.e. $\widehat{MD}$ the percentage change in MD and let's break down what happens when $\widehat{MD} > 0$. The impact of an increase in MD can be determined using the useful properties for growth rates presented in Technical Appendix A1.

On the one hand, increasing MD is in turn accompanied by a decrease in Q_0, the critical threshold above which abatement becomes socially worthwhile. Recalling that $Q_0 = \frac{P_a}{MD}$ and using the logarithmic differentiation properties, we find that the magnitude of the reduction is roughly equivalent to the increase in MD, i.e. $\hat{Q}_0 \cong -\widehat{MD}$. On the other hand, the rise in MD is associated by an increase in SMC, which is quite intuitive as pollution becomes more damaging, making optimal mixing more attractive. Recalling that $SMC = MD\sqrt{\frac{Q_0}{Q}}$ and using the same properties, we obtain that the percentage change of SMC for a given value of Q (i.e. a percentage change of Q equal to zero) is given by $\widehat{SMC}(if\ \hat{Q} = 0) \cong \widehat{MD} + \frac{1}{2}(\hat{Q}_0 - \hat{Q}) = \widehat{MD} + \frac{1}{2}\hat{Q}_0 = \widehat{MD} - \frac{1}{2}\widehat{MD} = \frac{1}{2}\widehat{MD}$. The optimal quantity at the social optimum will therefore decrease, because as the SMC curve shifts up, the point where this curve crosses the MNB^Q curve is associated with a smaller quantity on the horizontal axis (see figure C6.5).

Two remarks:

Lifting the ambiguity on $\hat{a}$ and $\hat{A}$. As the isoquant shifts in and the iso-cost curve becomes steeper, we know for sure that E will decrease, but the evolution of a and A is apparently ambiguous. We can in fact lift this ambiguity by total differentiation of the optimality condition $SMC = MNB^Q$. For that, we once again use the usual properties of growth rates denoting $\hat{x}$ (see Technical Appendix A1) to obtain $\widehat{SMC} = \widehat{MD} + \frac{1}{2}(\hat{Q}_0 - \hat{Q}) = \frac{1}{2}\widehat{MD} - \frac{1}{2}\hat{Q}$ on the one hand. $\widehat{MNB}^Q$ can be determined by recalling that $MNB^Q = \frac{20}{3} - \frac{4}{3}\sqrt{Q}$ and using the properties of growth rates of a sum and our knowledge of the initial optimal value of $Q_2^0 = \frac{81}{4} \Rightarrow MNB^Q(Q_2^0) = SMC(Q_2^0) = \frac{2}{3}$. This leads to $\widehat{MNB}^Q = \frac{20/3}{MNB^Q} \cdot 0 - \frac{4/3\sqrt{Q}}{MNB^Q} \cdot \frac{1}{2}\hat{Q} = -\frac{20/3 - MNB^Q}{MNB^Q} \cdot \frac{1}{2}\hat{Q} = -\frac{(20/3) - SMC(Q_2^0)}{SMC(Q_2^0)} \cdot \frac{1}{2}\hat{Q} = -\frac{(20/3) - 2/3}{2/3} \cdot$

$\frac{1}{2}\hat{Q} = -\frac{9}{2}\hat{Q}$ on the other hand. Applying $\widehat{SMC} = \widehat{MNB^Q}$ leads to $\frac{1}{2}\widehat{MD} - \frac{1}{2}\hat{Q} = -\frac{9}{2}\hat{Q} \Rightarrow 4\hat{Q} = -\frac{1}{2}\widehat{MD} \Rightarrow \hat{Q} = -\frac{1}{8}\widehat{MD}$. Total differentiation of conditions [2] in section 6.1.4 leads thus to $\hat{E} = \frac{1}{2}(\hat{Q} + \hat{Q}_0) = \frac{1}{2}\left[-\frac{1}{8}\widehat{MD} - \widehat{MD}\right] = -\frac{9}{16}\widehat{MD}$ and $\widehat{1+a} = \frac{1}{2}(\hat{Q} - Q_0) = \frac{1}{2}\left[-\frac{1}{8}\widehat{MD} + \widehat{MD}\right] = \frac{7}{16}\widehat{MD}$ so we know that a increases. Moreover, as $A = E^P - E = Q - E$, where $Q = Q_2^0 = \frac{81}{4}$, $E = \frac{9}{2} = E^0$ and $A = A^0 = \frac{63}{4}$, we can calculate that $\hat{A} = \frac{Q_2^0}{A^0}\hat{Q} - \frac{E^0}{A^0}\hat{E} = \frac{81}{63}\left(-\frac{1}{8}\widehat{MD}\right) - \frac{18}{63}\left(-\frac{9}{16}\widehat{MD}\right) = 0$, so we conclude that A remains constant. This is due to our ad hoc specification of the production function. Moreover, this is only true in linear approximation. As worksheet calculations can illustrate, in fact the level of abated emissions decreases slightly. The impact of shifting the isoquant and the iso-cost curve (solid orange and blue lines) can be observed in the following figure.

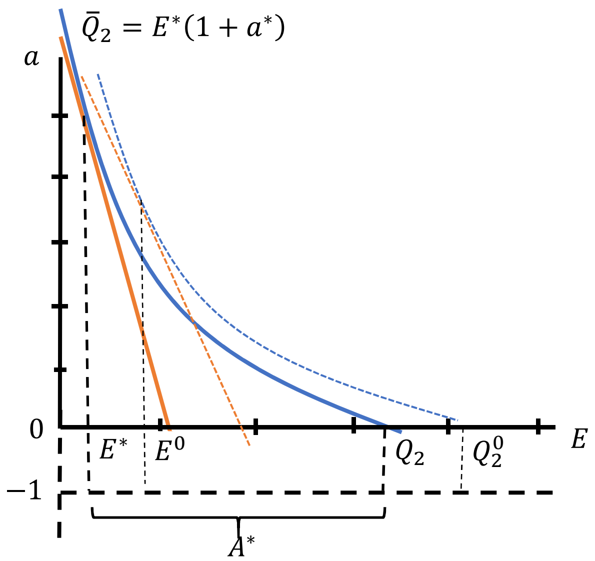

Figure C6.7: Impact of an increase in MD

Limit cases. As MD keeps on increasing, we can think of two possible limit cases illustrated by the stylized diagrams below. In case a), the MD is so high that the restriction-only optimal quantity (Q_3) becomes equal to the critical threshold for abatement activities (Q_0). In that case combining abatement with restrictions remains clearly worthwhile socially speaking, as the MNB^Q curve is systematically above the SMC schedule. In case b), the MD is even higher so that the restriction-only optimal quantity becomes zero (negative values make no economic sense) and, in the case drawn, the net social loss on the first units (from $Q = 0$ to Q_1 i.e. the pink area) is perfectly compensated by the net social gain on the last units (from $Q = Q_1$ to Q_2 i.e. the green area). In this limit case, the benevolent planer would just be indifferent between combining abatement with restriction at Q_2 or dropping production altogether.

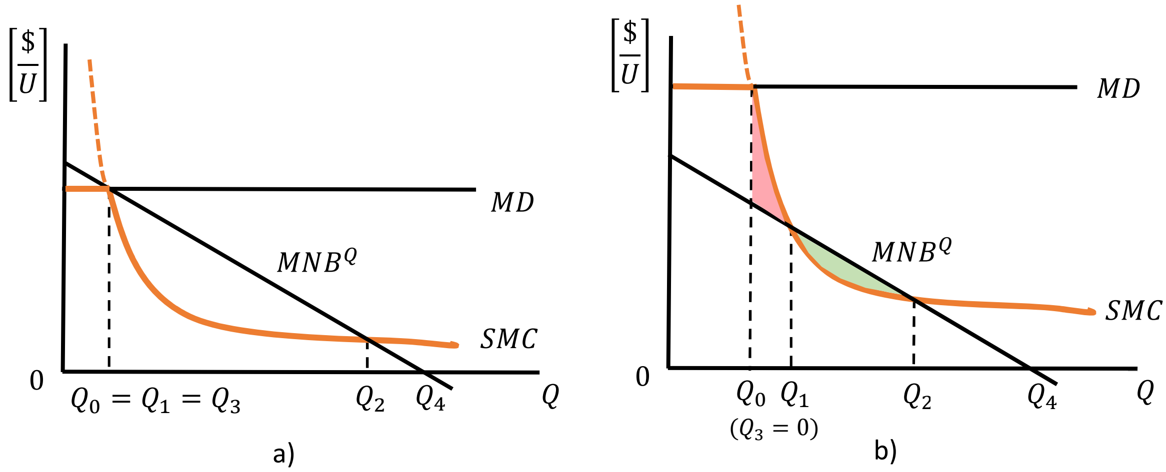

Figure C6.8: Limit cases of an increase in MD

Problem 6.4: Pollution control in Cleartown

Marginal benefit (MB): from the MD function, as $E = 30 - A$, one can write the MB of abatement function as $MB = 10(30 - A) - 50 = 250 - 10A$ for $5 < E < 25$ i.e. $5 < A < 25$ (and hopefully it is in this interval that the optimal abatement will locate, as confirmed below).

Marginal cost of total abatement (MAC): the efficient allocation of effort has to follow the principle of TCM (see chapter 1) i.e. $MAC_1 = a_1A_1 = MAC_2 = a_2A_2 = MAC$, where a_1 and a_2 are fixed coefficients (with different values depending on the question, see below). Rearranging this leads to $A_1 = \frac{MAC}{a_1}$, $A_2 = \frac{MAC}{a_2}$, so that $A = A_1 + A_2 = MAC\frac{(a_1+a_2)}{a_1a_2}$ or still $MAC = A\frac{a_1a_2}{a_1+a_2}$.

i. If $a_1 = 10$, $a_2 = 20$, then $MAC = A(20/3)$.
   Optimality condition (principle of TNBM): $MB = 250 - 10A = MAC = A(20/3)$ which implies $A *= 15$, $MAC = MB = 100$, $A_1^* = 100/10 = 10$ (so $E_1^* = 20 - 10 = 10$) and $A_2^* = 100/20 = 5$ (so $E_2^* = 10 - 5 = 5$).

ii. In that case, by chance, the authority has set the legal constraint equal to the efficient level, both in the aggregate and for each producer. So there is no welfare loss because both total abatement ($A^* = 15$) and individual abatement ($A_1^* = 10$ and $A_2^* = 5$) are equal to their optimal level. But this is so only because the largest emitter (producer 1) is also the most efficient one. If both producers would share the same MAC function, they should share the same abatement effort at the social optimum. This would mean an identical absolute abatement effort per producer which would be inconsistent with the same proportional abatement effort (50% rule).

To check that, if $a_1 = a_2 = 40/3$, then $MAC = A(20/3)$ (same aggregate MAC curve, but with different underpinings). TNBM principle: $MB = 250 - 10A = MAC = A(20/3)$ still implies $A *= 15$, $MAC = MB = 100$, but now $A_1^* = 100(3/40) = 7.5$ (so $E_1^* = 20 - 7.5 = 12.5$) and $A_2^* = 7.5$ (so $E_2^* = 10 - 7.5 = 2.5$). So the 50% reduction rule ($E_1^* = 10$ and $E_2^* = 5$) implies no welfare loss at the level of global abatement, which is optimal, but the effort is not efficiently shared. More precisely, with respect to optimal individual abatement levels, producer 1 (2) abates too much (too little) by 2.5 units. This translates into a wedge between $MAC_1 = 400/3$ and $MAC_2 = 200/3$ and generates a net welfare loss of $0.5((400/3) - (200/3))2{,}5 = 250/3$ (see dashed areas of diagram below). With respect to the total optimal abatement costs of $2(0.5)(100)(7.5) = 750$, this represents an extra cost of roughly 11% (1/9).

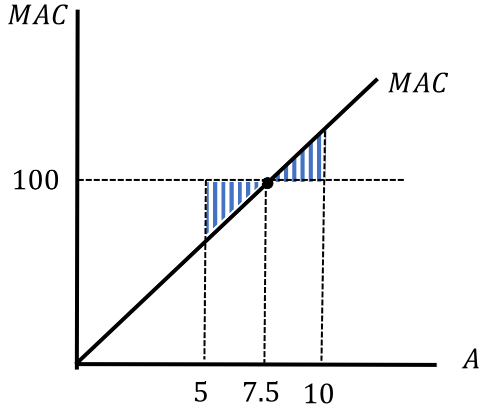

Figure C6.9: Marginal abatement cost and abatement effort

iii. If $a_1 = 10$, $a_2 = 20$, as we have seen that this case is efficient by chance, the MAC are equalized and there are no incentives to trade emission allowances between producers.

To check that, consider producer 1. Given the legal constraint, it has to abate 10, which means $MAC_1 = 100$. If the offered price for allowances is larger than 100, it would be willing to sell allowances (abate more) until the MAC equals the price i.e. its excess supply of allowances, ES_1, is given by $P = a_1(A_1^* + ES_1) = 10(10 + ES_1)$. If the price is lower than 100, it will be ready to buy allowances (abate less) until the MAC decreases to the price level, i.e. its excess demand of allowances, ED_1, is given by $P = 10(10 - ED_1)$. In fact, a negative excess supply is equal to a positive excess demand ($ES_1 = -ED_1$) and vice versa, so the two equations are equivalent. In the end, a single equation is sufficient to describe the willingness to trade by firm 1, e.g $P = 10(10 + ES_1) = 100 + 10ES_1$. It shows that the "indifference" price of firm 1 is 100 i.e. if the price is just equal to 100 firm 1 is neither willing to buy nor to sell allowances.

The same reasoning applied to firm 2 leads to $P = a_2(A_2^* + ES_2) = 20(5 + ES_2) = 100 + 20ES_2$. In other words, the indifference price is identical between the two firms, which are just happy like that and not willing to trade emission allowances (the implicit equilibrium price of allowances is indeed 100 but it does not materialize into an effective trade).

Things become more interesting if $a_1 = a_2 = 40/3$. In this case, the willingness to trade emission allowances is given by $P = (40/3)(10 + ES_1) \cong 133.33 + (40/3)ES_1 = 133.33 - (40/3)ED_1$ for firm 1 and $P = (40/3)(5 + ES_2) \cong 66.66 + (40/3)ES_2$ for firm 2. If the price locates in the [66.66; 133.33] interval, firm 1 is a consumer and firm 2 a supplier on the market for allowances. The situation is represented on the diagram below, with an equilibrium price of 100 and an equilibrium quantity of $ED_1 = ES_2 = 2.5$.

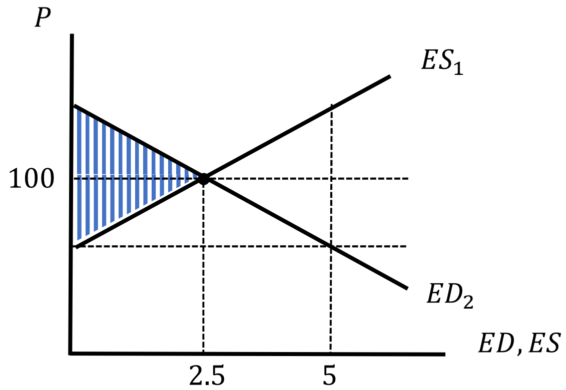

Figure C6.10: Excess supply and demand of allowances

In that case, allowing firms to trade allowances generates a net gain for society, which is equivalent to the extra cost already calculated at point ii. (the scale of the horizontal axis is different but the total shaded area is indeed identical between the two diagrams).

Problem 6.5: Cap&trade

As the authority wants to reduce emissions by 14 units, it sells 16 units to firms (and introduces prohibitive penalties so that firms respect allowances). Each firm will abate as long as the price is larger than its abatement cost, so we find the level of abatement by equalizing price (P) to the marginal abatement cost of each firm i.e. $P = 0.5 + 0.5A_1$, $P = A_2$ and $P = 1 + A_3$, from which we infer $A_1 = 2P - 1$, $A_2 = P$, $A_3 = P - 1$ (note that A cannot be negative, so $P > 0.5$ for 1 and $P > 1$ for 3). The demand for allowances in each case is simply obtained by $E_i = 10 - A_i$, $i = 1,2,3$ that is $E_1 = 11 - 2P$, $E_2 = 10 - P$ and $E_3 = 11 - P$. From that we obtain total demand $E = 32 - 4P$, and as supply is equal to 16, we obtain $P = 4$ when demand equals supply. So, the abatement efforts will be $A_1 = 7$, $A_2 = 4$, $A_3 = 3$. If a tax replaces the cap&trade system, it must be set equal to the equilibrium price (4).

Problem 6.6: demographic transition

i.

See the completed Excel file *problem_6.6.xlsx*

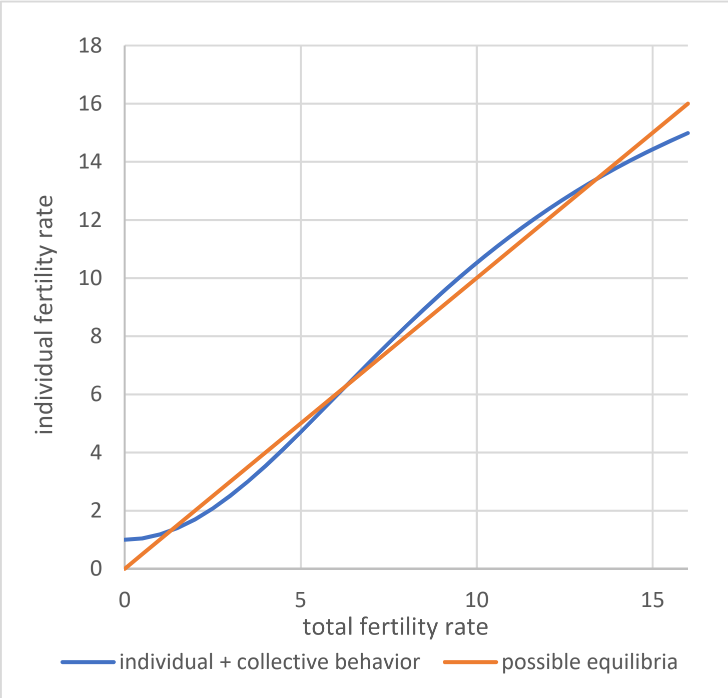

Figure C6.11: Fertility behavior

ii.

We can see on the diagram that there are three possible equilibria where the 45° line and the sigmoid curve cross. Using the Excel template (*problem_6.5_temp.xlsx*), we obtain $x_1 \cong 1.32$, $x_2 \cong 6.29$, $x_3 \cong 13.40$. The process is similar to the stylized case of lake eutrophication covered in chapter 4, where two equilibria are stable, while the last one is unstable. To understand this, recall that if the fertility rate of an additional woman is bigger than the average fertility rate of society, the average will increase. Conversely, if the fertility rate of an additional woman is smaller than the collective fertility rate, this will reduce the average fertility rate. x_2 is unstable, because if we locate at this point and consider an additional woman, the individual fertility rate will be above average, driving up the total fertility rate until x_3 is reached. Conversely, if we now consider the last additional woman just before point x_2, the individual fertility rate will be below average, so that the total fertility rate will fall until x_1 is reached. Using the same idea, we get that x_1 and x_3 are stable equilibria.

iii.

See the completed Excel file *problem_6.6.xlsx*

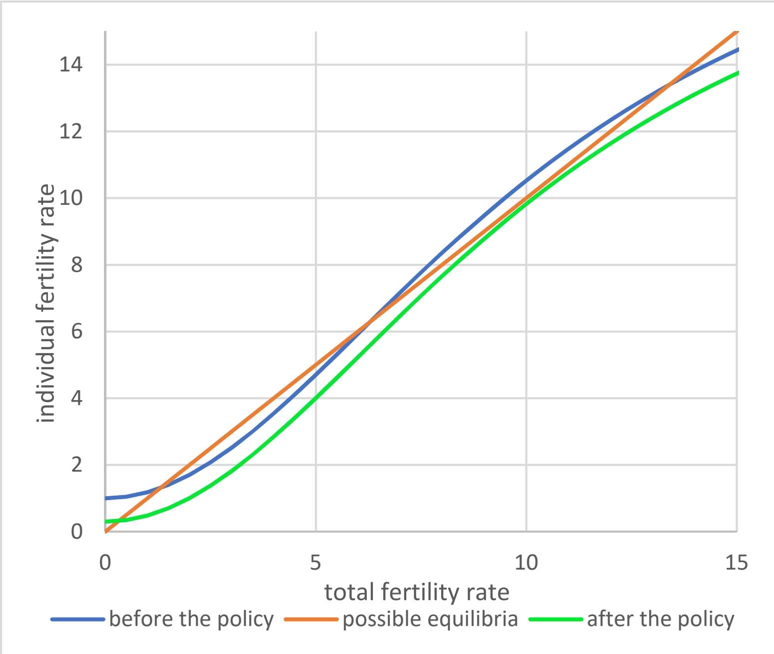

Figure C6.12: Impact of policy on fertility behavior

We initially locate at point x_3, where average fertility rate of the society is very high. An adequate policy will affect expectations regarding the number of children for a transitory period, so that every new woman entering the population wishes to have fewer children. This will shift the sigmoid curve downwards (green curve) and the average fertility rate will decrease. The policy should be sufficiently strong to shift the sigmoid curve below the 45° line in the relevant range of x values. The policy should be maintained until the average fertility reaches the threshold level at point $x_2 \cong 6.29$. Once this point has been reached ($x < x_2$), the desired number of children of an additional woman is below average. Therefore, even if the policy is abandoned and expectations return to the original sigmoid curve (in blue), the average fertility rate will progressively converge towards the low equilibrium level at point x_1.