Principles of sustainability economics: Extended correction guide

Chapter 5, problems 5.1 to 5.6

Problem 5.1: Energy use

Supply is given by production plus imports minus exports. Therefore, at the world-wide level, supply and production should be equal, but not necessarily so at the national level (supply is larger than production for net energy importers and mutatis mutandis for net energy exporters). Final consumption is necessarily smaller than supply, the difference being given by intermediate consumption (equal to roughly a third of final consumption at the world-wide level, most of it is represented by electricity plants).

Using figure 5.1, total energy supply shifts from a bit less than 400 EJ in 1997 to 618 in 2021, that is, an increase of at least 50%. For table 5.1, this implies that if the 1800-1997 period is extended to 2021, the "×35" multiplicative term should be replaced by a "×50" term (or 54.2 to get an exact 50% increase): $\frac{400}{35} \cong 11.4 \Rightarrow \frac{618}{11.4} \cong 54.2$.

Problem 5.2: Zero discount rate

We first need to calculate q_1^*: $P = 100 - q = 80 \Rightarrow q_1^* = 20$.

For the constant MEC case: $T^* = ceil(\bar{T})$ where $\bar{T} \equiv \bar{Q} \,/\, q_1^*$, so $\bar{T} = \frac{518}{20} = 25.9$. This means that $T^* = 26$ and during the transition (last) period, only 18 is extracted instead of 20.

For the increasing MEC case: such that i.e. $30 + \frac{25}{259}Q = 80 \Rightarrow \tilde{Q} = 518$, which leads to the same extraction length as for the constant MEC case.

The trajectories are those reported in figure 5.7 in the notes.

Problem 5.3: Increase in MEC_s

i. Zero discount rate

If the MEC_s increases for a given extraction length, this means that the MUC of the first period with respect to the last period of extraction (T^*) increases, as $MUC = MEC_s - MEC$. This means total marginal cost (MEC plus MUC) increases, while the MB schedule is kept unchanged, so the quantity extracted per period has to decrease.

Depending on the magnitude of the increase in MEC_s, an additional extraction period (or several of them) will be necessary to exhaust the resource. Thus, the optimal extraction length tends to increase (it remains stable if the total decrease in extracted quantity is less or equal to the consumption of the renewable in the last period).

At the limit, when the MEC_s tends towards the choke price (P_c), quantity per period becomes infinitesimally small while the extraction length becomes infinity long (but the product between the two keeps on being equal to $\bar{Q}$). This remains valid when $MEC_s \geq P_c$: the resource is so scarce that it is **infinitely spared** (this seems a bit extreme, but remember we are just describing an efficiency condition, and with zero discount rate).

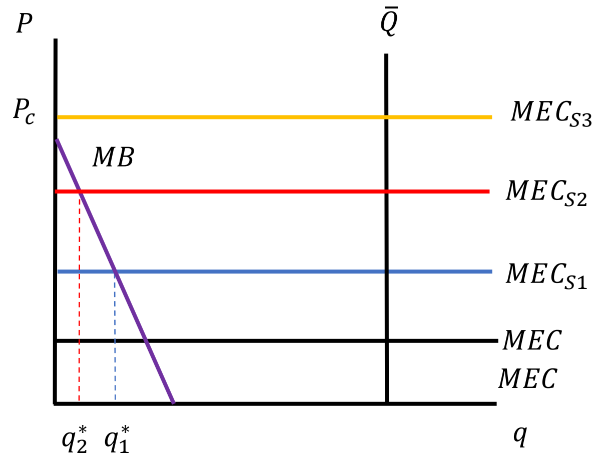

Figure C5.1: Impact of a progressive increase of the MEC_S on the extracted quantity q

ii. Strictly positive discount rate

Let us consider the first period. Vis à vis the last period of extraction, the MUC is given by $\frac{MEC_S - MEC}{(1+r)^{T^*}}$. For a fixed T^*, this quantity unambiguously increases when MEC_S increases, so the extracted quantity decreases. The same reasoning applies to subsequent periods. Here again, depending on the magnitude of the increase in MEC_S, an additional extraction period (or several of them) may be necessary to exhaust the resource and the optimal extraction length tends to increase, as in the case of the zero discount rate. The diagram next page represents the stylized impact of an increase of the MEC_S on the optimal quantity and price trajectories.

However, when the discount rate is positive, the implied increase in the optimal extraction length is smaller than in the zero discount rate case. Why? Because an extra year *decreases* the present value of the MUC of the last period of extraction (because of the discounting factor at the denominator of the equation in the above paragraph). In other words, when the discount rate is positive, adding an extra year always softens the sparing effort for later generations.

This refinement changes the limit properties of the optimal behavior. Even if $MEC_S \geq P_c$, there will always be some (finite but sufficiently large) value of T^* such that the present value of the MUC is smaller than $P_c - MEC$, which allows for a strictly positive extraction at period 1. So the "infinitely long" extraction length result disappears when the discount rate is strictly positive.

Figure C5.3 represents the impact of a progressive increase in the MEC_S, with an increase in the extraction length and a decrease in extracted quantities per period, and the limit case achieved when $P_c = MEC_S$ and the extracted quantity is equal to zero at the last period (orange trajectory). Note that whatever the case, total quantity extracted remains equal to $\bar{Q}$ (that means that the integral between $t = 0$ and T^* below each one of the coloured curves at the left hand side is equal to $\bar{Q}$).

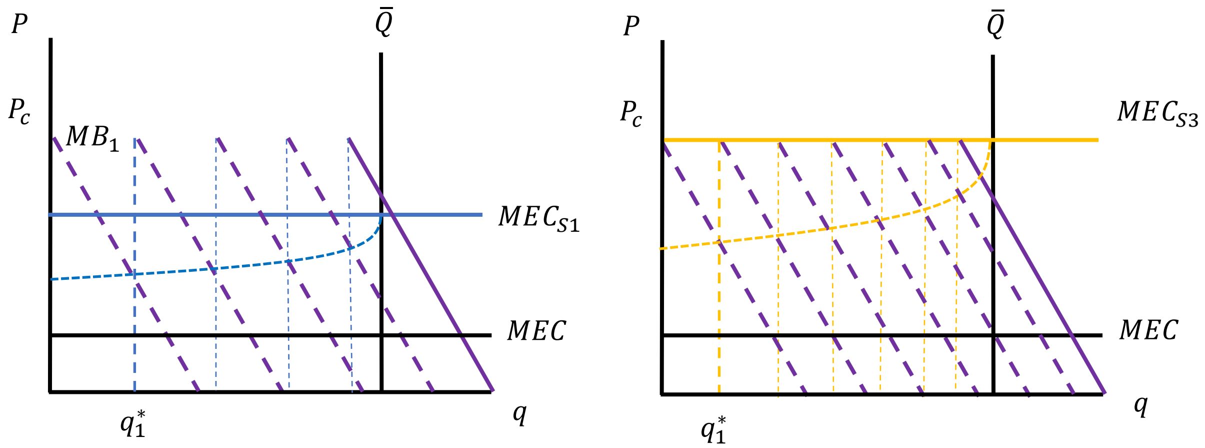

Figure C5.2: Impact of a progressive increase of the MEC_S on the extracted quantity

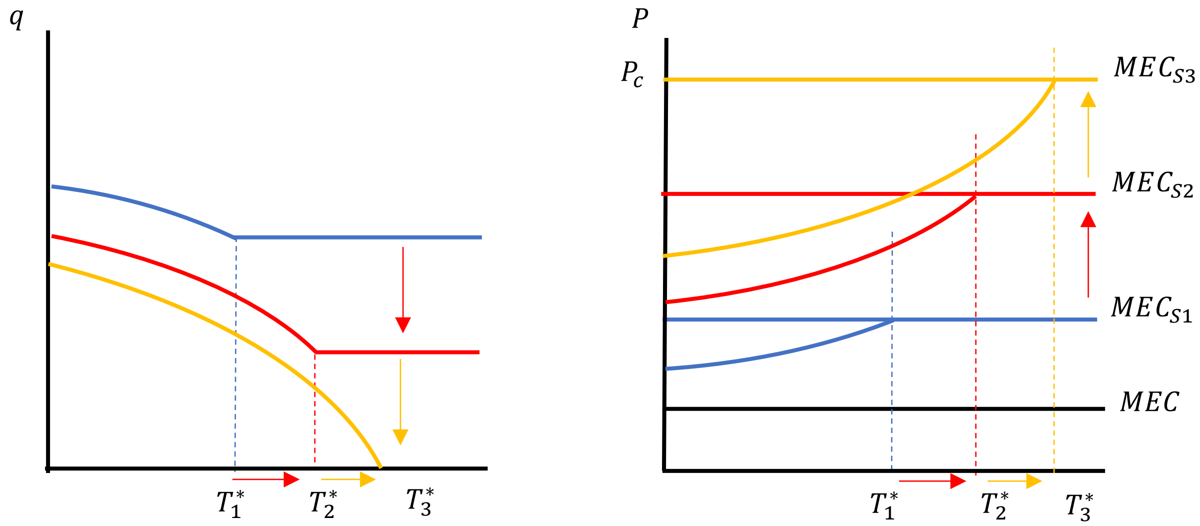

Figure C5.3: Impact of a progressive increase of the MEC_S

Problem 5.4: Depletable substitute

If the best alternative after the exhaustion of the (initial) depletable resource ($\bar{Q}_1$) is still another depletable resource (available in quantity $\bar{Q}_2$, with $MEC_2 > MEC_1$) the transition to the renewable backstop (or abstinence) is further delayed in time.

The shape of the optimal trajectories can be identified in two steps. To simplify, we will assume that $\bar{Q}_1 = \bar{Q}_2$ and a renewable substitute is available with a marginal extraction cost (MEC_S) intermediate between MEC_2 and the choke price ($MEC_2 < MEC_S < P_c$).

The first step is to analyze separately the two depletable resources, as represented by figure C5.4. As the alternative depletable is costlier to extract, it starts with a larger price in period 1, which leads to a larger optimal extraction length ($T_2^* > T_1^*$).

Figure C5.4 (and straightforward intuition) suggests that the cheapest resource should be exploited first. This would shift the whole price trajectory for resource 2 to the right. However, for resource 1, the MUC of the last period will not be equal to segment a anymore (as would be the case if there was only resource 1), but to segment b (i.e. at the start of resource 2 extraction). In other words, as the transition to the next resource is made at a lower marginal extraction cost (MEC_2 instead of MEC_S), the lower MUC leads to a larger extracted quantity in period 1 (and subsequent periods), and therefore to a smaller extraction length ($T_1^* < \tilde{T}_1^*$).

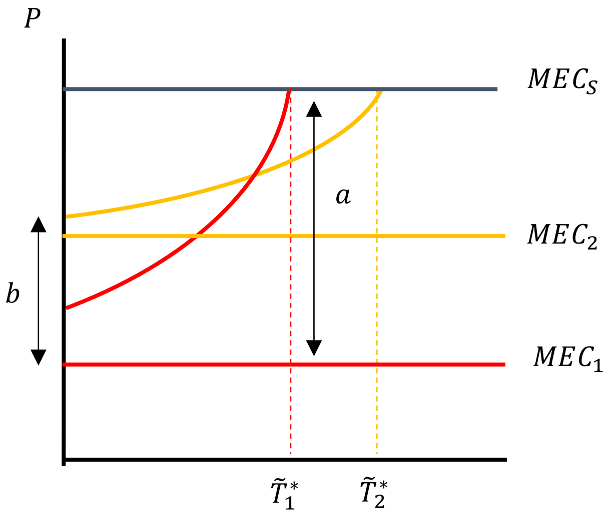

Figure C5.4: Comparing two depletable resources ($\bar{Q}_1 = \bar{Q}_2, MEC_2 > MEC_1$)

The combined stylized trajectories appear in Figure C5.5, with quantities on the left panel and prices (and marginal costs) on the right panel. The price trajectory now presents two convex sections, one for each depletable resource, until it reaches the plateau at the MEC_S level. Conversely, the quantity trajectory presents two concave sections.

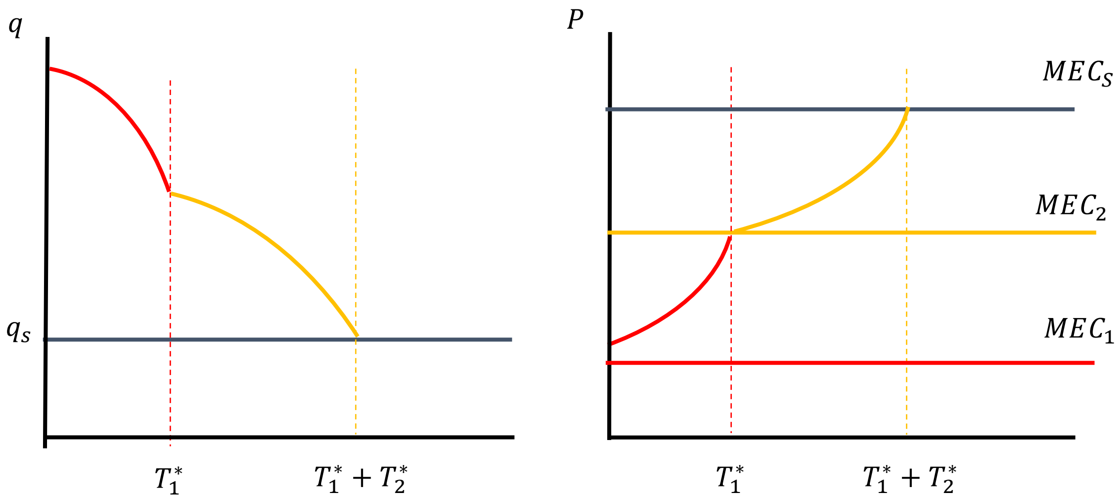

Figure C5.5: Optimal sequence of extraction trajectories with two depletable resources

Problem 5.5: Increase in the discount rate

If the discount rate increases, the present value of the MUC decreases, which tends to favor larger extraction today and a decrease in the extraction length. Overall, the concavity/convexity of the price or quantity trajectories increases, which illustrates a further departure from the reference case of a zero discount rate (flat line trajectories). The same backward reasoning can be applied as in the course, starting from period T^*_{-1}, then T^*_{-2} etc. In the constant MEC case (Figure C5.6), the present value of the MUC in period T^*_{-1} is given by $\frac{MEC_S - MEC}{(1+r)}$ which is a decreasing function of r. In the increasing MEC case (Figure C5.7), the present value of the MUC in period T^*_{-1} is given by $\frac{(MEC_S - MEC)\cdot Q_{T^*_{-1}}}{(1+r)}$, i.e. also a decreasing function of r. The same principles apply for further periods back in time.

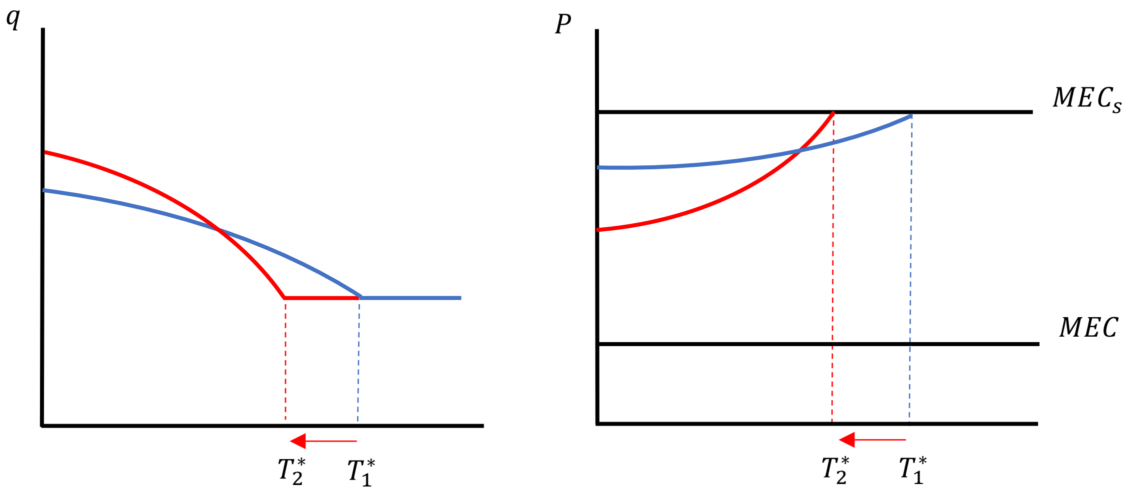

Figure C5.6: Impact of an increase in the discount rate - constant MEC case

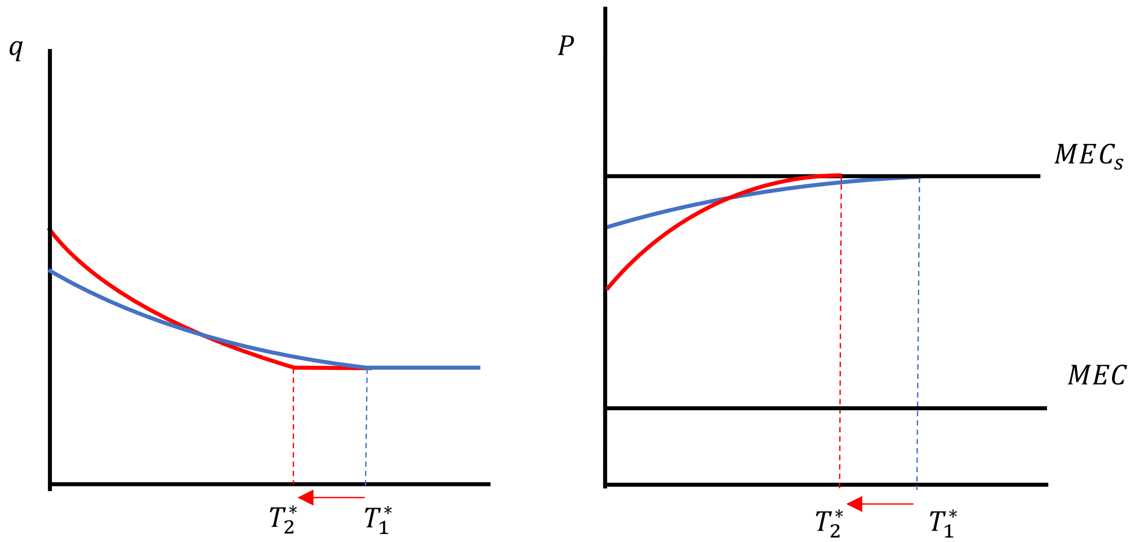

Figure C5.7: Impact of an increase in the discount rate - increasing MEC case

Problem 5.6: Import vulnerability

See the excel file *problem_5.6.xlsx* for the detailed calculations + beware that the consumption tax affects only the price paid by consumers, not the price received by producers.

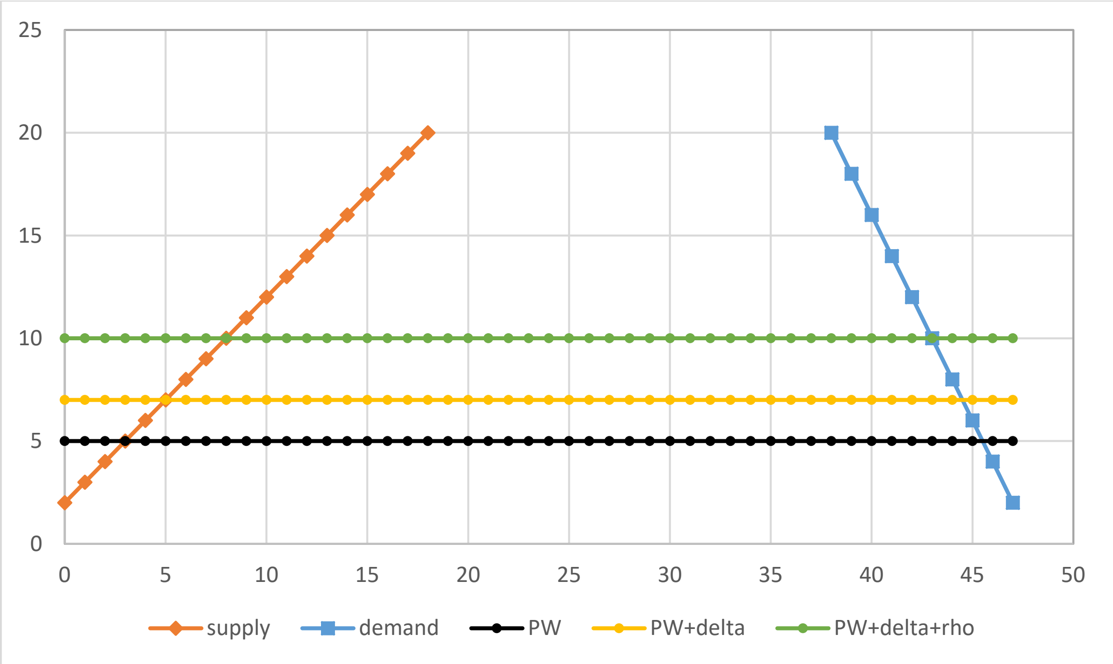

Figure C5.6: A stylized national market of an oil importing country

Net social gains, by increasing policy relevance (two externalities need two instruments):

i. Tariff $t_1 = \rho = 2$ +6
ii. Tariff $t_2 = \rho + \Delta = 5$ +6.25
iii. Tariff $t_1 = \rho = 2$ and consumption tax $t_3 = \Delta = 3$ +8.25

The highest social gains are obtained by the last option, which is more efficient because it addresses two different externalities with two distinct policy instruments.