Principles of sustainability economics: Extended correction guide

Chapter 4, problems 4.1 to 4.5

Problem 4.1: Sustainable harvesting

i. MSY is obtained when $\frac{dG}{dS} = r - 2r\frac{S}{\bar{S}} = 0 \Rightarrow S = \frac{\bar{S}}{2} = 50 \Rightarrow G = (0.02)(50)\left[1 - \frac{1}{2}\right] = 0.5$

ii. $G - H = 0$ with $G = (0.02)S\left[1 - \frac{S}{100}\right] = 0.02S - 0.0002S^2$ and $H = \theta(MSY) = 0.5\theta$ where $\theta = 0.19$ or $\theta = 0.51$ or $\theta = 0.75$. Multiplying by 50 and regrouping we obtain the following condition for the steady state level of S:

$$0.01S^2 - S + 25\theta = 0$$

The solutions are given by:

$$\frac{1 \pm \sqrt{1-\theta}}{0.02} = 50\left(1 \pm \sqrt{1-\theta}\right)$$

Selecting the largest value in each case gives $S = 95, 85, or\ 75$ if $\theta = 0.19, 0.51\ or\ 0.75$ respectively.

iii. See excel file *problem_4.1.xlsx*

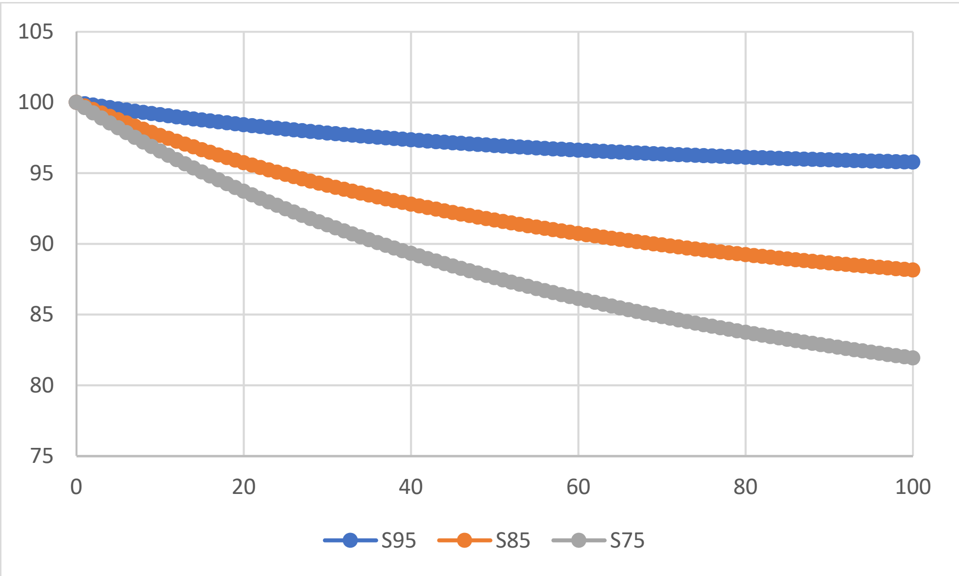

Figure C4.1: Harvesting case

We can check in the excel file that in order to achieve 90% of the gap between steady states (of 5, 15 and 25 respectively), one has to wait for around 125, 151 or 194 years respectively = the larger the gap, the longer the period, but less than proportionately i.e. the larger the pressure, the faster the exhaustion of the resource.

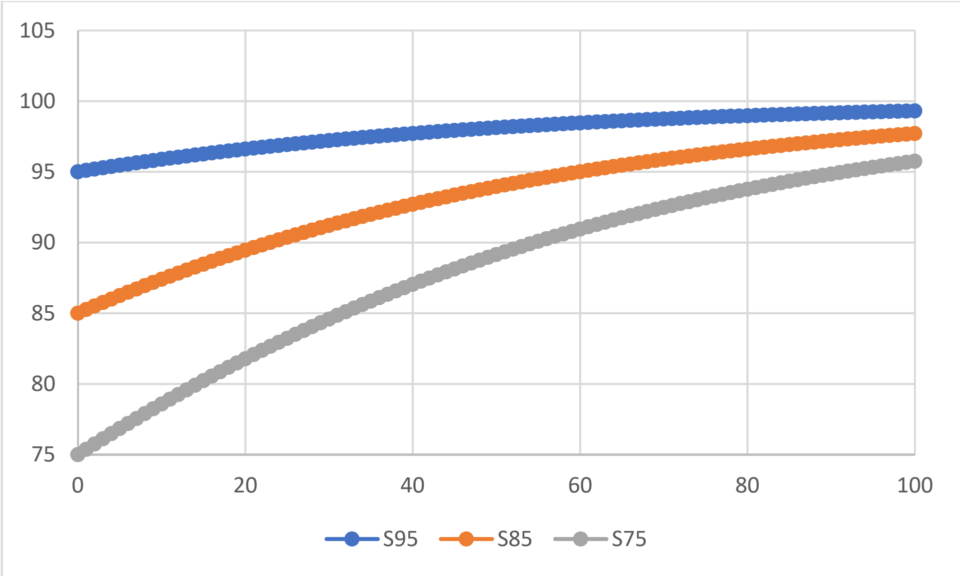

Figure C4.2: Restoring case

We can check again in the excel file that in order to achieve 90% of the gap between steady states (of 5, 15 and 25 respectively), one has to wait for around 117, 122 or 128 years respectively = the larger the gap, the longer the period, but again less than proportionately i.e. the larger the pressure on the resource has been, the faster the restauration rate of the resource.

Note that this symmetry in behavior between exploiting and restoring the resource is not necessarily verified in all ecosystems, as commented in the last section of this chapter on hysteresis.

Problem 4.2: Steady-state relationships

With $G = rS\left[1 - \frac{S}{\bar{S}}\right]$ and $H = qES$, the condition for the steady state, $G = H$, leads to $r\left[1 - \frac{S}{\bar{S}}\right] = qE$, which can be re-arranged as: $S(E) = \bar{S}\left[1 - \frac{q}{r}E\right]$. Re-introducing this expression in the harvest function, one obtains the following parabolas: $H(E) = q\bar{S}\left[E - \frac{q}{r}E^2\right]$
$TB(E) = PH(E) = Pq\bar{S}\left[E - \frac{q}{r}E^2\right]$.

Problem 4.3: Economic shock

See the diagram. The TC curve becomes flatter. In the case of the open access regime (points B and B'), this leads to a larger effort level, a smaller harvest, and a scarcity rent that remains equal to zero. This means that everybody in the community (i.e. those who fish and those who don't) just earn the wage rate, which has decreased, so total income decreases.

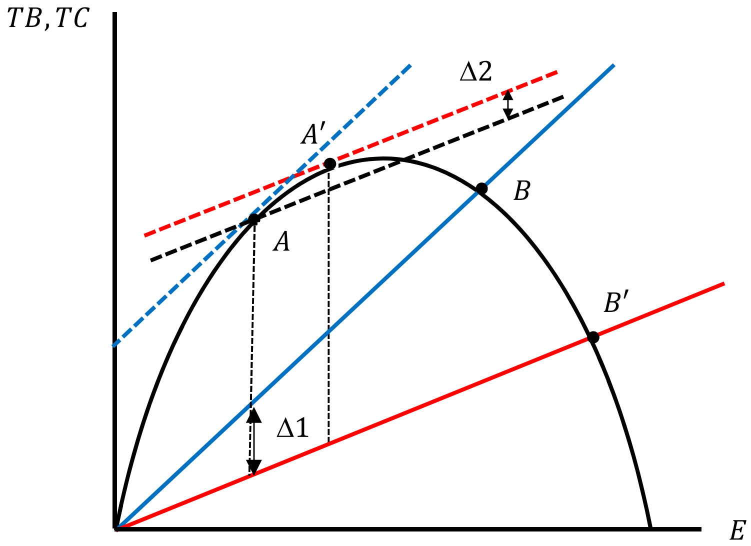

Figure C4.3: Economic shock

In the case of efficient harvesting (points A and A'), the effort increases and the harvest also increases. The scarcity rent increases for two reasons: i. for a given E, TC is smaller (segment $\Delta 1$) and ii. at the new wage rate, the effort level adjusts to its new optimum level (segment $\Delta 2$). So, the welfare of those who fish definitely increases. However, for those who do not fish, total income decreases. So, in the end, the net impact on the whole community depends on the share of fishermen (if it is sufficiently large, total income increases).

Problem 4.4: Numerical simulation

i.

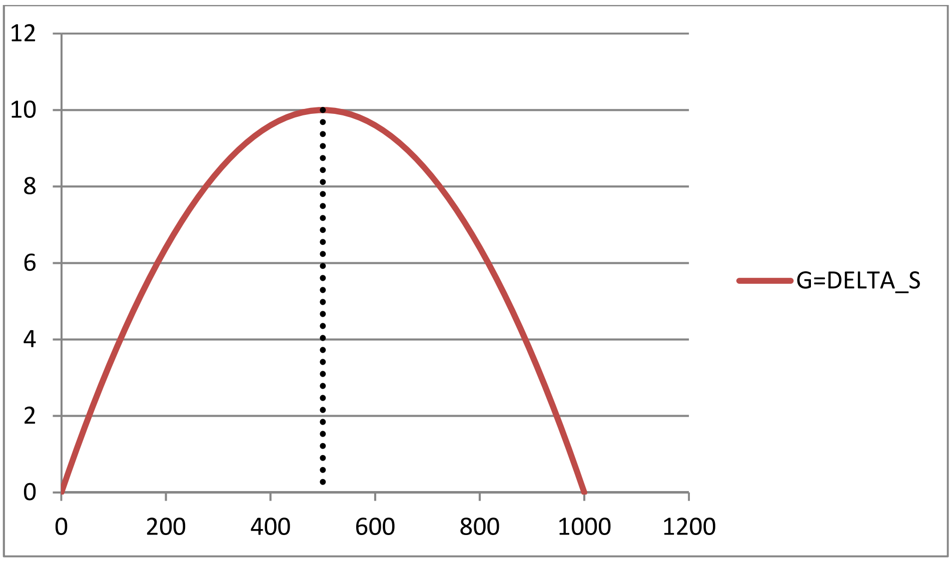

Figure C4.4: Gordon-Schaefer model

Maximum sustainable yield obtained at the hump of the curve i.e. when:

$$\frac{dG}{dS} = 0 \Rightarrow 0.04\left(1 - \frac{S}{\overline{S}}\right) - \frac{1}{\overline{S}}0.04S = 0 \Rightarrow S = \frac{1}{2}\overline{S} \quad [C4.1]$$

Precisions: as $G = \Delta S = 0.04S\left(1 - \frac{S}{\overline{S}}\right)$ we can write: $G = uv$ where $u = 0.04S$, $v = 1 - \frac{S}{\overline{S}}$, and then apply the derivative of a product of functions i.e. $(uv)' = u'v + uv'$

Which leads to $S = 500$ for $\overline{S} = 1000$, implying $G = 10$ (in thousand units).

ii. The steady state is obtained when $G - H = 0$, i.e. $0.001ES = 0.04S\left(1 - \frac{S}{\overline{S}}\right)$, from which we obtain $0.001E = 0.04\left(1 - \frac{S}{\overline{S}}\right)$, so $E = 40\left(1 - \frac{S}{\overline{S}}\right)$ and as $\overline{S} = 1000$, $E = 40\left(1 - \frac{S}{1000}\right)$, so:

$$S = 1000 - 25E \quad [C4.2]$$

Plugging back this equation into $H = 0.001ES$, one obtains the amount harvested, in thousand units. If we multiply that by 1 (i.e. one thousand francs per unit, or 1 million francs per thousand units), we obtain total benefit, in million francs, i.e. :

$$TB = 0.001E(1000 - 25E) = E - 0.025E^2 \quad [C4.3]$$

It can be checked that the maximum is obtained for $E = 20$, i.e. $S = 500$, which is consistent with the maximum sustainable yield identified at point i.
Precision: $TB' = 1 - 0.05E$, so setting $TB' = 0$ (maximum) leads indeed to $E = 20$.

Regarding total cost, it is obtained by multiplying E (in thousand days of fishing) by 0.5 (thousand francs per fishing day), i.e., in million francs:

$$TC = 0.5E \quad [C4.4]$$

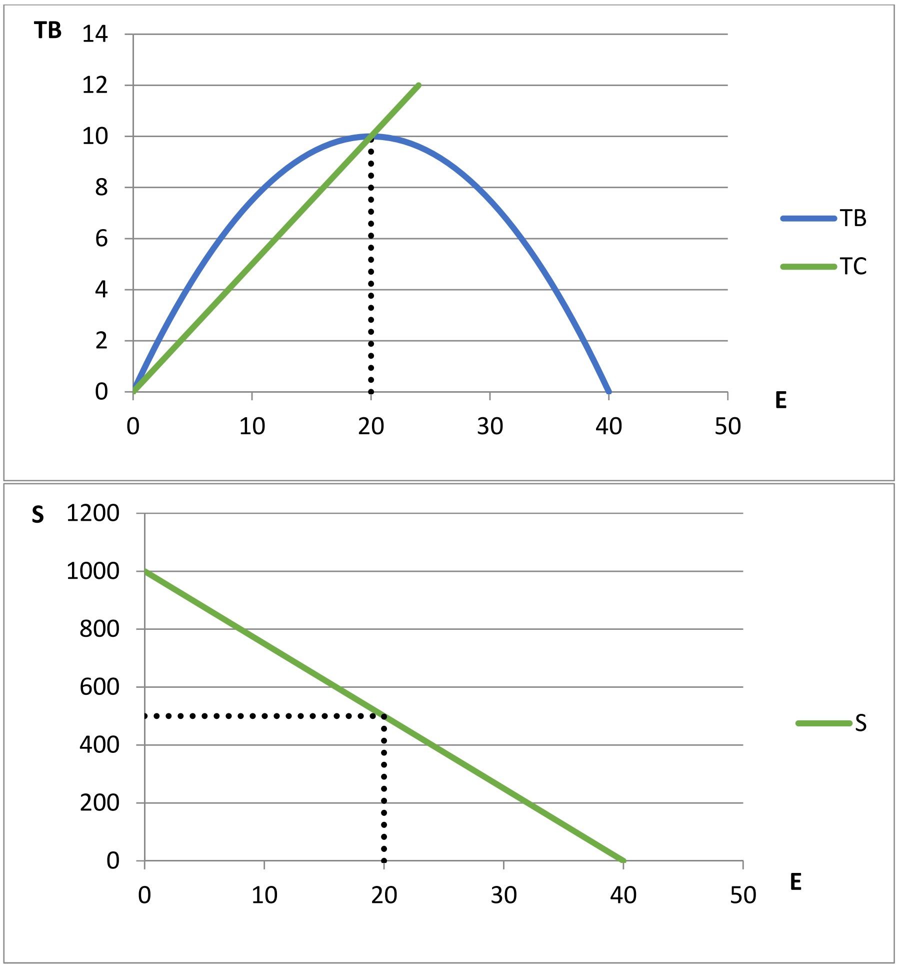

Figure C4.5: Total benefit, total cost, and fish stock

iii.

The efficient effort is obtained when $dTB\ /\ dE = 1 - 0.05E = dTC\ /\ dE = 0.5$, which leads to $E^e = E = 10$ and (using [2]), $S^e = 750$

The open-access fishing level is obtained when $TB = TC$ i.e., after a few manipulations, when $E^c = 20$ and (using [2]), $S^c = 500$.

The social welfare loss is given by $TB(E^e) - TC(E^e) = 7.5 - 5 = 2.5$ million francs, and the overfishing intensity is $E^c\ /\ E^e = 2$.

In this very special case, the open-access fishing effort corresponds to the maximum sustainable yield (this cannot be generalized).

iv.

First, in the case of a decrease in the wage rate, the new total cost is: $TC_2 = 0.25E$. As for point iii the efficient fishing effort is obtained for $dTB\ /\ dE = dTC\ /\ dE$, leading to $1 - 005E = 0.25$ and thus

$E^e = 15$, $S^e = 625$. As for point iii the open-access fishing effort is obtained for $TB = TC$, leading to $E^c = 30$ and $S^c = 250$.

As opportunity costs are lower, a larger fishing effort is accepted, and social welfare is optimized for a larger harvest. The stock of fishes (S) decreases strongly following this reduction in labor costs.

Second, in the case of an increase in the fish price: $P_2 = 1.5$, TB becomes $TB_2 = 0.0015E(1000 - 25E) = 1.5E - 0.0375E^2$. Thus, $E^e = 13.333$, $S^e = 666.675$, $E^c = 26.667$ and $S^c = 333.325$. The increase in the price of fish increases TB, and with it the fishing effort, which leads to a decrease in the fish stock.

Whatever the case, the intensity of overfishing is unchanged ($E^c \,/\, E^e = 2$).

v.

One must find t (in thousand francs per day) such that the new total cost schedule with the tax ($(0.5 + t)E$) intersects the total benefit schedule at the vertical of E^e, that means when $(0.5 + t)E^e = E^e - 0.025(E^e)^2$, from which we obtain, replacing E^e by 10, $t = 0.25$ thousand francs per day, i.e. an equivalent ad valorem tax of 50%.

vi.

One must first find the efficient effort in this case, denoted by E^{app}. It is obtained by equalizing the net marginal revenue of the decrease in effort, i.e. $-(dTB \,/\, dE - dCT \,/\, dE) = -(1 - 0.05E - 0.5) = -0.5 + 0.05E$, with the corresponding marginal cost, i.e. 0.25. One obtains $E^{app} = 15$. Then, if one repeats the same procedure as for point v., but for that effort level, i.e. $(0.5 + t)E^{app} = E^{app} - 0.025(E^{app})^2$, one obtains $t = 0.125$, i.e. an equivalent ad valorem tax of 25%. Intuitively, as this policy is more costly than at point v, it must be applied with a lower intensity.

Problem 4.5: Lake eutrophication

See the completed Excel file *problem_4.5.xlsx*

i. As the phosphorus concentration P increases, recycling R increases, but the relationship is not linear, i.e. the recycling rate $\frac{\partial R}{\partial P}$ is not constant.

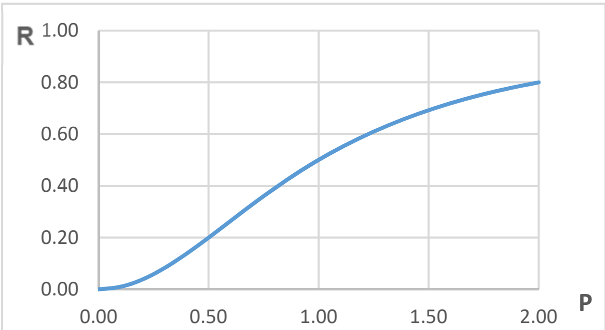

Figure C4.6: Evolution of $R(P)$

R is influenced by the hypolimnion, i.e. the degree of oxygenation of the bottom layer of the lake. With low levels of P, when it increases, R increases slowly, because the hypolimnion has enough oxygen. At intermediate levels of P, oxygen levels drop, boosting up R. Once the inflexion point has been reached, at high levels of P, R slows again because the lake bottom constantly lacks oxygen.

To determine the inflexion point of $R(P)$, we compute the first derivative $\frac{dR}{dP} = \frac{2P(1+P^2)-P^2(2P)}{(1+P^2)^2} = \frac{2P}{(1+P^2)^2}$ and determine its maximum value: $\frac{d^2R}{dP^2} = \frac{2(1+P^2)^2-2P\cdot 2(1+P^2)\cdot 2P}{(1+P^2)^4} = \frac{2(1+P^2)-8P^2}{(1+P^2)^3} = \frac{2-6P^2}{(1+P^2)^3}$ and evaluating to 0: $\frac{d^2R}{dP^2} = 0 \Rightarrow \frac{2-6P^2}{(1+P^2)^3} = 0 \Rightarrow 2 - 6P^2 = 0 \Rightarrow P^2 = \frac{1}{3} \Rightarrow P = \frac{1}{\sqrt{3}}$ and $R = \frac{1/3}{4/3} = 0.25$. Note that P has to be positive, so we only have 1 solution.

To find λ^c_{high}, which corresponds to the slope at the inflexion point, we have to substitute $P = \frac{1}{\sqrt{3}}$ in $\frac{dR}{dP}$, which gives $\frac{2\frac{1}{\sqrt{3}}}{\left(1+(\frac{1}{\sqrt{3}})^2\right)^2} \cong \frac{3\sqrt{3}}{8}$.

ii. The steady state equilibrium values of P are obtained when $G(P) = 0 \Rightarrow -S + R = 0: -\lambda P + \frac{P^2}{1+P^2} = 0 \Rightarrow \lambda P(1 + P^2) - P^2 = 0 \Rightarrow P[\lambda(1 + P^2) - P] = 0 \Rightarrow P[\lambda P^2 - P + \lambda] = 0 \Rightarrow$ $P_1 = 0$ and P_2, P_3 depend on the value for λ.

Use the formula in Technical Appendix A3. Quadratic equations to find $P = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm \sqrt{\Delta}}{2\lambda}$ with $\Delta = b^2 - 4ac = 1 - 4\lambda^2$.

When $\lambda = 0.8$: $\Delta = 1 - 4(0.8)^2 = -1.56 < 0$, so we have only one stable equilibrium at $P_1 = 0$ and no other equilibrium when $\lambda > \lambda^c_{high}$.

G is negative and downward-sloping, so the resilience capacity of the lake is sufficiently large for it to remain stable regardless of pollution levels.

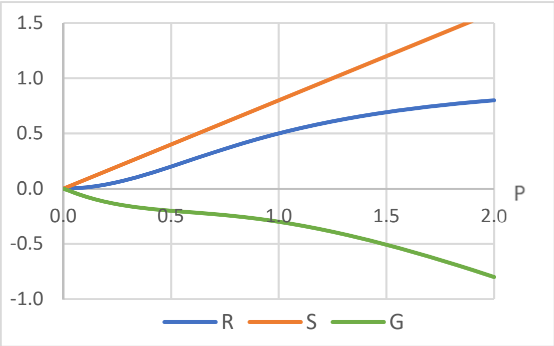

Figure C4.7: Stock growth when lambda = 0.8

iii.

One stable equilibrium at $P_1 = 0$ and another equilibrium when $\lambda = 0.5$: $\Delta = 1 - 4(0.5)^2 = 0 \Rightarrow$ $P_2 = \frac{1}{2(0.5)} = 1$.

$\lambda = 0.5$ corresponds to the low critical level λ^c_{low} and we can observe that the $G(P)$ curve presents a hump for intermediate levels of P. Unlike P_1, P_2 is an unstable equilibrium, so, over time, the lake will return to its original condition thanks to the gradual reduction in phosphorus.

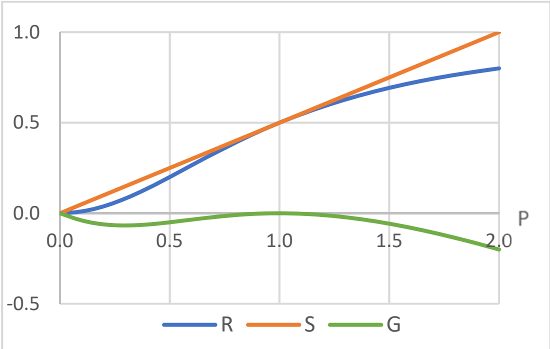

Figure C4.8: Stock growth when lambda = 0.5

iv.

One stable equilibrium at $P_1 = 0$ and another equilibrium when $\lambda = 0.4$: $\Delta = 1 - 4(0.4)^2 = 0.36 \Rightarrow$ $P_2 = \frac{1-\sqrt{0.36}}{2(0.4)} = 0.5$ and $P_3 = \frac{1+\sqrt{0.36}}{2(0.4)} = 2$

For low values of $\lambda < \lambda_{low}^c$, there are 3 equilibria. P_1 and P_3 are stable and P_2 is unstable. Over time, for low levels of $P < P_2$, the lake will return to its pristine conditions. However, for levels of $P > P_2$, the phosphorus concentration will gradually increase to reach its steady state level P_3. This means that the resilience capacity of the lake is too weak, leading to a phosphorous concentration that remains high.

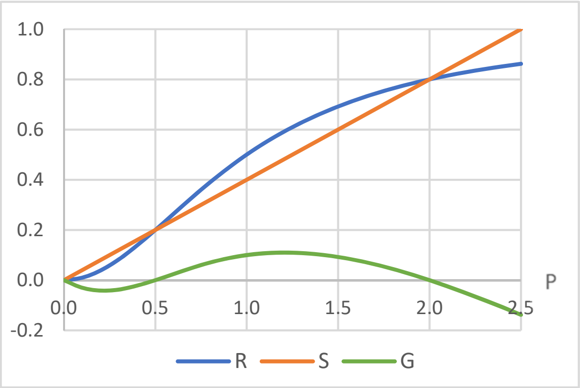

Figure C4.9: Stock growth when $\lambda = 0.4$

v.

When the resilience capacity of the lake is large, i.e. $\lambda > \lambda_{high}^c$, the P concentration gradually sinks over time, allowing the lake to restore its initial pristine conditions. This is true regardless of the magnitude of phosphorus discharges.

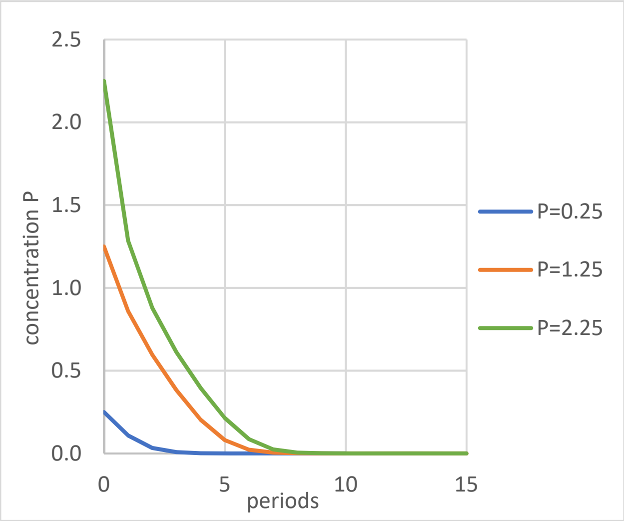

Figure C4.10: Evolution of P over time when $\lambda = 0.8$

When the resilience capacity of the lake is intermediate, i.e. $\lambda_{low}^c < \lambda < \lambda_{high}^c$, the P concentration gradually declines over time and the lake recovers its initial conditions. However, compared with a high resilience capacity, the natural restoration process takes longer for more severe discharges. The process slows down until the hump of the $G(P)$ curve is reached (convex part of the trajectory) and then accelerates again (concave part of the trajectory).

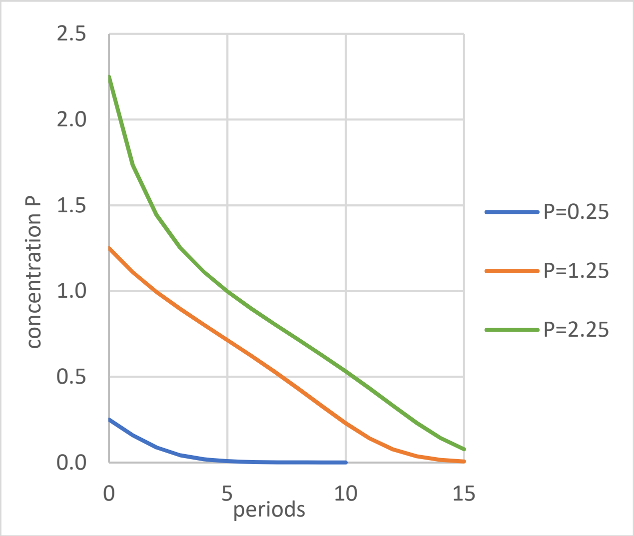

Figure C4.11: Evolution of P over time when $\lambda = 0.6$

When the resilience capacity of the lake is small, i.e. $\lambda < \lambda_{low}^{c}$, the $P(t)$ trajectories depend on the magnitude of the shocks. Natural restoration will only occur in the case of small P discharges. For medium and large shocks, the P concentration will tend towards the steady state equilibrium $P_3 = 2$ determined in point iv.

Thus, it is impossible for the lake to return to its initial pristine conditions if the resilience capacity of the ecosystem is weak, leading to permanent eutrophication.

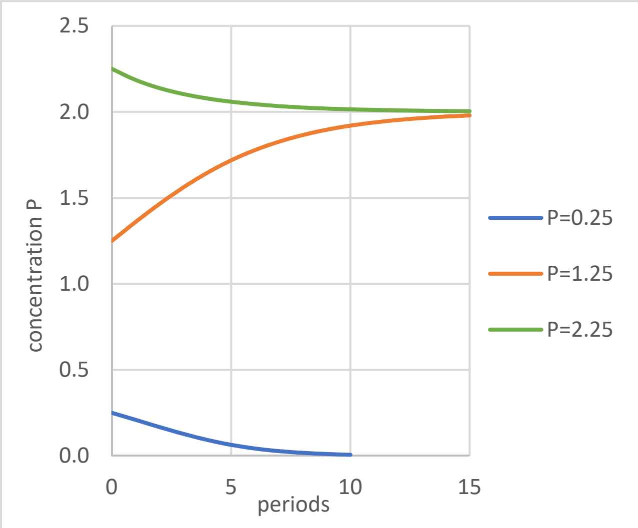

Figure C4.12: Evolution of P over time when $\lambda = 0.4$