Principles of sustainability economics: Extended correction guide

Chapter 3, problems 3.1 to 3.5

Problem 3.1: Efficiency, Equity and Transfer

i. zero discount rate

Diagram: see *problem_3.1.xlsx*

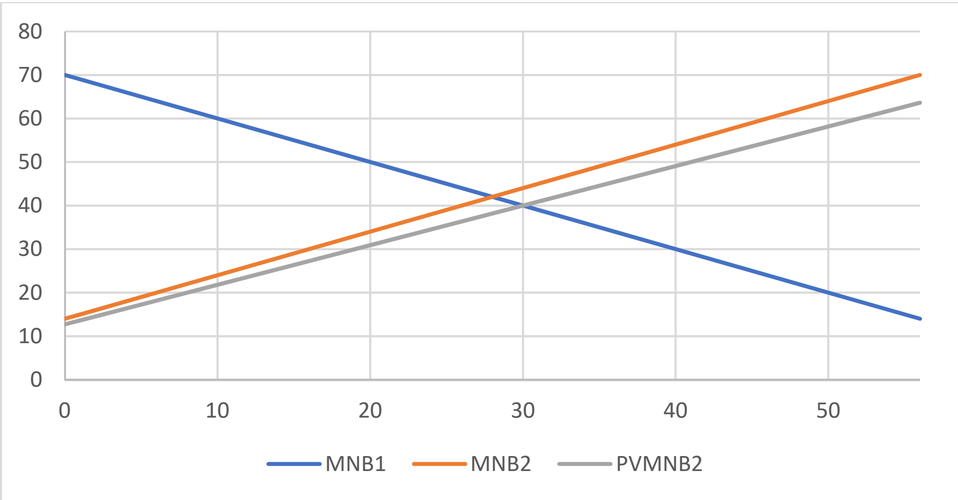

Figure C3.1: Problem 3.1: The allocation box

The efficient and fair allocation is obtained equalizing marginal net benefits:

$$MNB = MB - MEC = 100 - q - 30 = 70 - q$$

Equalization MNB: $70 - q_1 = 70 - q_2 = 70 - (56 - q_1) = 14 + q_1 \Rightarrow 56 = 2q_1 \Rightarrow q_1 = q_2 = 28$

$TNB_1 = TNB_2 = 0.5[70 + 42]28 = 1568$

ii. discount rate of 10%

Efficient allocation:

Equalization between the present value of marginal net benefits: $MNB_1 = PVMNB_2 \Rightarrow 70 - q_1 = \frac{14+q_1}{1.1} \Rightarrow 63 = 2.1q_1 \Rightarrow q_1 = 30, q_2 = 26$

In current values:

$TNB_1 = 0.5[70 + (70 - q_1)]q_1 = 0.5[70 + 40]30 = 1650\ (+82)$

$TNB_2 = 0.5[70 + (70 - q_2)]q_2 = 0.5[70 + 44]26 = 1482\ (-86)$

$\Rightarrow$ from fair (point i.) to efficient allocation, $TNB_1 + TNB_2$ decreases by -4

In present values:

Fair allocation: $PVTNB_1 = 1568,\ PVTNB_2 = \frac{1568}{1.1} = 1425.45 \Rightarrow$ total: 2993.45

Efficient allocation: $PVTNB_1 = 1650,\ PVTNB_2 = \frac{1482}{1.1} = 1347.27 \Rightarrow$ total: 2997.27

$\Rightarrow$ from fair (point i.) to efficient allocation, $PVTNB_1 + PVTNB_2$ increases by 3.82

iii. transfer

There are two equivalent ways to answer that question. First, one can reason conceptually, noting that the allocation is efficient i.e. the extracted quantities are as in point ii but there is a transfer from generation 1 to generation 2 so that generation 2 is at least as well off as generation 1 in terms of **current** values. Therefore, the transfer must satisfy two conditions: 1) it must perfectly compensate generation 2 from its loss starting from the fair (but inefficient) allocation of point i (i.e. -86 in current value) and 2) it must share fairly the net social gain from point i to point ii. Condition 1) means that, in present value, the transfer must include $\frac{86}{1.1} = 78.18$. Condition 2) means that a share λ of the net gain of 3.82 (see point ii must be transferred to generation 2 (with an interest rate of 10%) so that what is left for generation 1 in current value of period 1 ($3.82(1-\lambda)$) is equal to what is obtained by generation 2 in current value of period 2 ($3.82\lambda(1.1)$). This leads to $3.82(1-\lambda) = 4.202\lambda \Rightarrow 8.022\lambda = 3.82 \Rightarrow \lambda \cong 47.62\%$, which indeed corresponds to a transferred share of $\lambda = 1/2.1$. The total transfer, in **present value**, is thus given by $78.18 + (0.4762 \cdot 3.82) \cong 80.0$.

The alternative way is to use the following diagram:

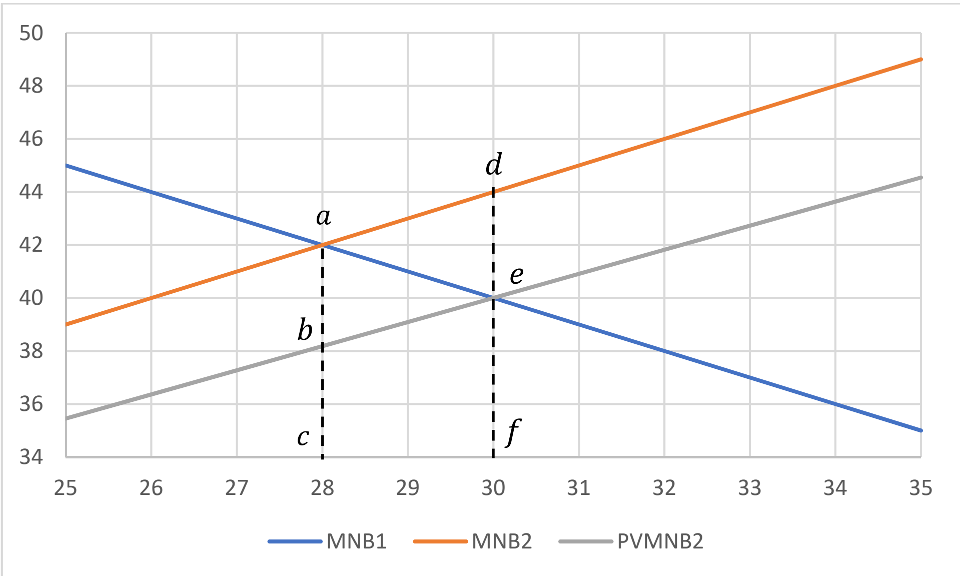

Figure C3.2: Problem 3.1: An intergenerational transfer

NB_1: points c and f are supposed to be on the horizontal axis (vertical axis has been cut to make the diagram clearer)

NB_2: the vertical value for point b (i.e. segment bc) is given by $\frac{70-q_1}{1.1} = \frac{70-28}{1.1} = 38.18$

First, generation 1 has to transfer $befc$, which is equal to area $adfc$ in future value (by construction) to compensate generation 2 for its loss.

Second, generation 1 still wins area aeb. She has to transfer part of it so that the two generations earn the same in current value. In other words, the condition for the share to be transferred, denoted by λ must be:

$$\lambda(aeb)(1.1) = (1-\lambda)(aeb) \;\Rightarrow\; 2.1\lambda = 1 \;\Rightarrow\; \lambda = \frac{1}{2.1}$$

Thus, in total, generation 1 has to transfer: $becf + \left(\frac{1}{2.1}\right) aeb = 0.5(38.18 + 40) \cdot 2 + \frac{0.5(42-38.18)\,2}{2.1}$ $= 78.18 + 1.81 = 80$

Generation 2 will receive $80(1.1) = 88$

So, after the transfer, $TNB_1 = 1650 - 80 = 1570$ and $TNB_2 = 1482 + 88 = 1570$. In other words, with respect to the "fair" allocation ($q_1 = q_2 = 28$), each country wins 2.

Problem 3.2: Well-being discount rate (WBDR) and consumption discount rate (CDR)

Recap: the CDR is given by $r = \delta + |\eta|g$ where δ is the WBDR, $|\eta|$ the absolute value of the elasticity of marginal utility with respect to consumption and g is the growth rate of consumption per capita.

As $u = \ln(c)$ we have $u' = 1/c = c^{-1}$ which means $\eta = \frac{du'}{dc}\frac{c}{u'} = -c^{-2}c^2 = -1$.

Substituting we obtain: $r = 0.01 + 0.015 = 0.025$ i.e. r is 2.5%.

Problem 3.3: Sustainability theorem

see the completed excel file *problem_3.3.xlsx* (or its template if you want to realize part of the calculation steps by yourself). The sheet "Settings" allows you to change the parameters of the problem (in particular the initial stock of both types of capital) and reports the basic results obtained in the corresponding simulation sheets. Simulations corresponding to the initial situation are reported in the sheet "base", while the other sheets, as their name indicates, correspond to changes in the capital stock (either infinitesimal or more consequential).

The logic of the model is represented by figure C3.3 below. The circular flow of income between firms (which produce Q on the basis of three factors of production: man-made capital, K_1, natural capital, K_2 and labor, L) and households (which receive Y, consume C and save S, which is used to finance investment in either man-made capital, I_1, or natural capital, I_2).

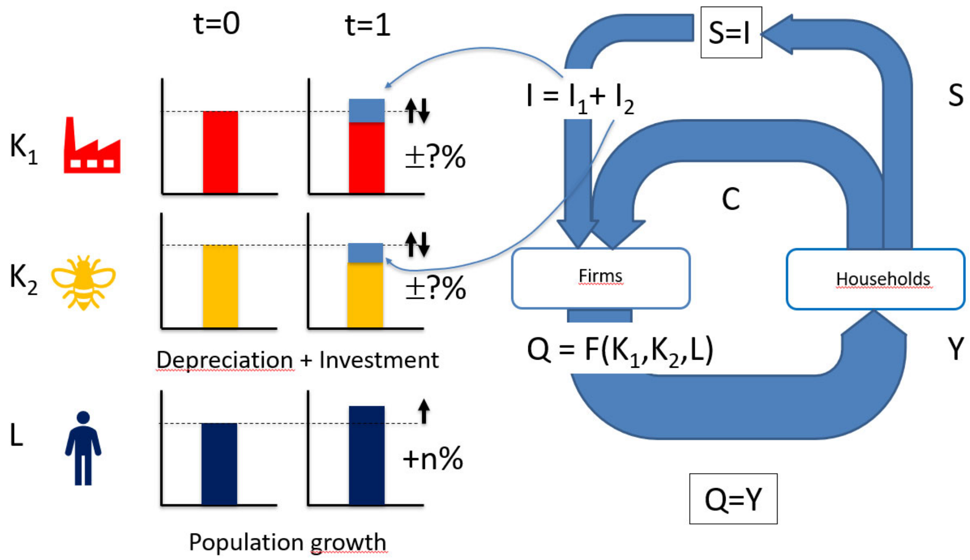

Figure C3.3: Problem 3.3: Income growth, savings and factor accumulation

In the first column on the left-hand side (t=0) appear the initial endowments of each factor (all equal to 1 to simplify). The second column contrasts the evolution in the next period (t=1). On the one hand, population is assumed to grow at a constant positive rate, n, so that the dark blue bar in column t=1 becomes bigger. On the other hand, both capital stocks tend to decrease due to depreciation (represented by the smaller red or yellow bars) and increase through investment (represented by the additional blue bars which come on top), so that the sign of their net growth is generally ambiguous.

Note that the same logic applies also to both problem 3.3 (where $Y = Q = N^{\alpha}K^{1-\alpha}$ and $K = K_1^{\beta}K_2^{1-\beta}$) and problem 3.4 (where $Y = Q = N^{\alpha}K^{1-\alpha}$ and $K = \left[\beta K_1^{\rho} + (1-\beta)K_2^{\rho}\right]^{1/\rho}$).

i. Pure depreciation would lead to $K_1 = K_2 = 1 - 0.02 = 0.98$. Given that overall savings are equal to $S = 0.2Y = 0.2$ we have an investment of $0.9 \cdot 0.2 = 0.18$ in K_1 and $0.1 \cdot 0.2 = 0.02$ in K_2. Thus, the final capital stock levels are given by $K_1 = 0.98 + 0.18 = 1.16$ and $K_2 = 0.98 + 0.02 = 1.0$. It is clear that the economy has the potential to avoid a decrease in both capital stocks, and does manage it at the end of the first period (man-made capital increases while natural capital remains just constant). However, savings allocation is biased, putting maximum investment effort on man-made capital.

ii. As K_1 increases by 16% and K_2 stays constant, and both types of capital enter the aggregation function with equal weight, it is intuitive that the aggregate capital stock increases somewhere between 16% and 0% (e.g. by 8%.) Applying the growth rules (see the Technical Appendix) leads indeed to $\widehat{K} \cong \beta\widehat{K_1} + (1-\beta)\widehat{K_2} = 0.5(0.16) + 0.5(0) = 0.08$. As $\widehat{N} = 0.01$, applying the same rules, we obtain $\widehat{Y} \cong \alpha\widehat{N} + (1-\alpha)\widehat{K} = 0.5(0.0.1) + 0.5(0.08) = 0.045$. As investment is directly proportional to savings and income, the investment effort for each capital type increases by 4.5%. As K_1 has increased by 16%, so has the decrease in this asset due to depreciation as it is based on a fixed rate of 2%, resulting in a higher total depreciation amount. Thus, man-made capital is expected to increase, but at a smaller rate than at the end of the first period. Oppositely, as natural capital has remained stable while investment has increased (through the increase in income), the growth rate of natural capital becomes positive. This pattern of convergence between the growth rates of each capital type can be expected to persist over time. The K_1/K_2 ratio becomes larger, as K_1 still increases at a larger rate than K_2. Moreover, the growth rate decrease of K_1 is larger than the growth rate increase of K_2, so that the growth rate of the total capital stock, K, comes down. Eventually, as illustrated by the diagrams in the "base" sheet of the excel file, in the very long run, these **endogenous** growth rates all converge towards the value of the unique **exogenous** growth rate, namely $\widehat{N} = 1\%$.

   This is what is called the **steady state** of the economy, with all variables K_1, K_2, K and Y growing at the same rate. In this situation, consumption per capita ($c = 0.8Y/N$) remains constant. In this long run situation, the element of the sum in the definition of intergenerational wellbeing (V) decreases, because the numerator grows at 1%, while the denominator grows at 2%. As a consequence, as illustrated by figure C3.4 below (see sheet "base" of the excel sheet), if one plots V as a function of the number of periods, a convergence towards a plateau appears.

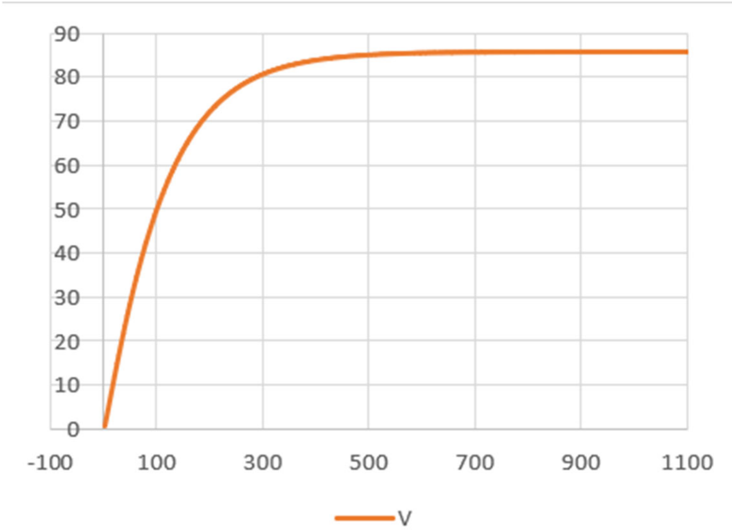

Figure C3.4: Problem 3.3: Intergenerational wellbeing as a function of time

iii. The worksheet can be used to simulate the impact on V of a **very small (infinitesimal)** change in the initial endowment of a given type of capital. Intuitively, we can expect a smaller price for man-man capital, because due to the bias in the allocation of savings, natural capital becomes scarcer over time and is not perfectly substitutable by man-made capital. Indeed, using 1100 periods to estimate V_t and calibrating the simulation with a change of 0.0001 for each type of capital, it can be checked that one obtains a price of around 2.2 for man-made capital and 6.3 for natural capital (see cells B18 and B19 of the sheet "settings"). The accounting price of natural capital is higher due to the fact that it becomes scarcer over time and is not perfectly substituted by man-made capital.

Using these accounting prices, one can calculate the value of inclusive wealth, W_t, and verify the sustainability theorem. More specifically thereafter, following **arbitrary changes** in initial endowments for both types of capital (e.g. dK=0.01 as in cell H9 of the "Settings" sheet of the worksheet, i.e. 100 times bigger than the infinitesimal changes), one can check that the same value is approximatively obtained by considering either the change in V_t or genuine investment (i.e. the change in W_t), thus proving numerically the sustainability theorem. As can be checked in cells I14 to I17 of the "Settings" sheet, the relative error is less than 1%.

Problem 3.4: Limited capital substitutability

see the completed excel file *problem_3.4.xlsx* (or its template if you want to realize part of the calculation steps by yourself)

In comparison with problem 3.3, we see that the value of V drops by around 20%, becoming roughly equal to 68.37 (cells B14-B16 of sheet "Settings"). This is intuitive. As the two types of capital are less substitutable ($\rho < 0$), the fact that natural capital is growing at a lower rate than man-made capital cannot be compensated as easily as in problem 3.3, which penalizes overall welfare. This also translates into a lower accounting price of man-made capital (1.21) and a larger price for natural capital (8.49). As a consequence, the ratio between the two is now equal to 7, while it was less than 3 in the Cobb-Douglas case.

By adjusting ρ to make it even smaller (cell B9 of the "Settings" sheet), we can observe more disparity between the two prices. For instance, with $\rho = -5$, the price for natural capital (9.48) becomes almost 25 times larger than the price for man-made capital (0.37). At the other extreme, if the two capital types become perfectly substitutable ($\rho \to 1$ i.e. $\sigma \to +\infty$), their accounting price becomes identical (equal to 3.44). As mentioned in the chapter, this makes inclusive wealth sensitive to the limited substitutability between capital types.

Problem 3.5: Environmental Kuznets Curve (EKC)

i. Using the approximative rules for growth rates one obtains:

$$\hat{E} = \hat{P} + \hat{y} + \varepsilon_{T,y}\hat{y} = \hat{P} + \hat{y}(1 + \varepsilon_{T,y})$$

where $\varepsilon_{T,y}$ is the elasticity of T with respect to y, which may be either positive or negative.

Interpretation: in general, to obtain a decrease in world emissions, one of the three terms at least must be negative, and sufficiently so that it more than compensates the two other ones. Now as the second and third term can be regrouped it seems to simplify but in fact a complication comes with the sign of $\varepsilon_{T,y}$. Three situations may arise:

- if $\varepsilon_{T,y} > 0$: Impossible to decrease emissions while maintaining both demographic ($\hat{P} > 0$) and economic growth ($\hat{y} > 0$). Either P must decrease (less population), or y must decrease (degrowth), or both.
- if $-1 < \varepsilon_{T,y} < 0$: similar to the previous case, although the net impact of $\hat{y}$ is less important (thus, if $\hat{y}$ remains positive, the required decrease in P is less severe, but if it is $\hat{P}$ which remains positive, the required decrease in y must be more pronounced).
- if $\varepsilon_{T,y} < -1$: Emissions may decrease while keeping both P and y increasing provided $\hat{y}$ and $|\varepsilon_{T,y}|$ are sufficiently large.

ii. We answer in two steps:

Step 1: Analytically (i.e. irrespective of the value of the parameters)

Fist, we analyze emission intensity i.e. $T(y)$. We put its derivative equal to zero:

$\frac{\partial T(y)}{\partial y} = 2ay + b = 0 \Rightarrow y^* = -b/2a$

As $\frac{\partial^2 T(y)}{\partial y^2} = 2a < 0$ we know that the function is concave, which is a necessary condition for a maximum (i.e. y^* is the point where $T(y)$ reaches a local maximum). Thus, the condition for emission intensity (i.e. kg per dollar) to decrease with y is that $y > y^* = -b/2a$.

Second, emissions per capita are given by $yT(y) = ay^3 + by^2 + cy$. Equalizing the derivative to zero gives: $3ay^2 + 2by + c = 0$. Applying the usual formula for solving quadratic equations (see Technical Appendix A1), the solutions are:

$$y^{**} = \frac{-2b \pm \sqrt{\Delta}}{6a}$$

Where $\Delta = 4b^2 - 12ac > 4b^2$ (remember that $a < 0$) which means $\sqrt{\Delta} > 2b$. Thus, only the solution with $-\sqrt{\Delta}$ makes economic sense (y cannot be negative) and we know that it will be larger than $-\frac{4b}{6a} = -\frac{2b}{3a} = \frac{4}{3}y^*$.

In other words, for emissions per capita to decrease with y we need $y > y^{**}$ (since $yT(y)$ is concave) which is equal to $\frac{4}{3}y^*$ plus an additional term which is a positive function of $|a|$ and c (and equal to zero if $c = 0$).

Step 2: Numerical part: see excel file *problem_3.5.xlsx* and diagrams below

Preliminary discussion: are we close to reality?

The average GDP per capita at the world-wide level in 2022 was approximately 13'000 USD per person (https://www.imf.org/external/datamapper/NGDPDPC@WEO/OEMDC/ADVEC/WEOWORLD). Thus, if we have $y_0 = 13$, and use the indicated values for parameters (either $a = -1, b = 40, c = 0$ or $a = -1, b = 20, c = 260$), this leads to emissions per person (in kg/person) of $y_0 T(y_0) = -13^3 + 40 \cdot 13^2 = 4563$kg i.e. around 4.6 ton of CO_2 per person. Multiplying by world population of around 8 billion persons leads to $E \cong 36'504$ billion kg i.e. 36.5 billion tons (or $GtCO_2$), which is not far from recognized estimates of 36.1 Gt CO_2 for 2022 (e.g. https://carbonmonitor.org/). In other words, even if the parameters have been arbitrarily chosen for the sake of the explanation, they represent realistic orders of magnitude for 2022.

The two cases look quite different even if they lead to the same initial values (see diagrams and beware of the different vertical scales). In case a), the world has still to reach the turning point in terms of emission intensity ($y_0 < y^*$). In case b), this turning point has already been passed ($y_0 > y^*$), which means that emission intensity is already decreasing, but emission intensity per capita is still increasing ($y_0 < y^{**}$). Thus, we can call case a) the "pessimistic" perspective and case b) the "optimistic" perspective.

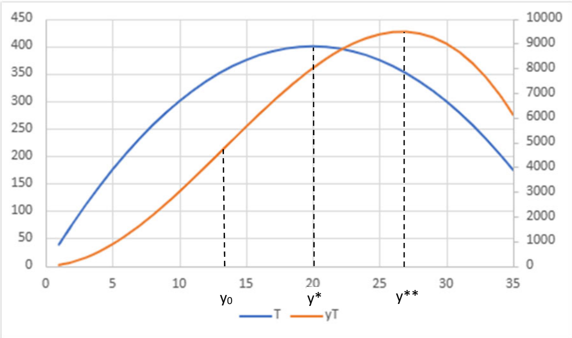

Figure C3.5: Problem 3.5 – set a)

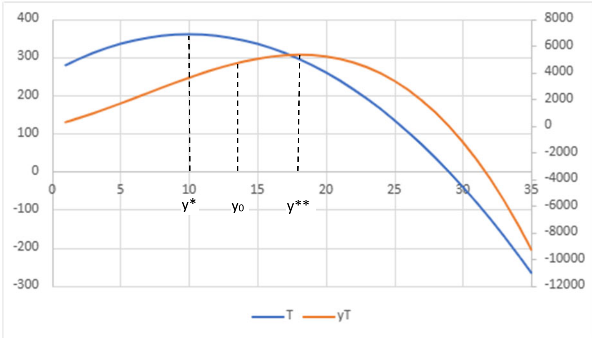

Figure C3.5: Problem 3.5 – set b)

Which case is closer to reality? The evidence reported in figure 17 in chapter 3 suggests emission intensity is decreasing since decades. Thus, we should rather adopt the optimistic perspective. Nevertheless, keep in mind that this remains a very stylized exercise, so we keep the two cases as a comparison basis (a more rigorous way to address the issue would be to use econometric techniques to select the specification which can best reproduced the observed data, as illustrated by the paper referenced in the chapter).

iii. Prepare for bad news.

First, plug in the values of the parameters into the analytical results to obtain the critical values given by the following table (see excel file for calculations)

	y* [thousand USD]	y**[thousand USD]
case a) - pessimistic view	20	26.66
case b) - optimistic view	10	18.12

Second, starting from $y_0 = 13$, we use the condition $y_t = y_0 1.015^t$ to find out how long it would take to get to a future critical value of y_t (either y^* or y^{**}).
Taking logs gives: $\ln(y_t) = \ln(y_0) + t ln(1.015)$ and re-arranging:

$$t = \frac{\ln(y_t/y_0)}{\ln(1.015)}$$

Using the excel sheet we obtain the following approximative results:

	years to reach y*	years to reach y**
case a) - pessimistic view	29	48
case b) - optimistic view	n.a.	22

Even if we adopt the optimistic perspective, this is probably too long to avoid climatic catastrophes.

Third, even if gloomy, the above calculations are still too optimistic, because in the present century, world population is still expected to increase. According to the UN population data, world population

will still grow till around 2080 (see diagram, but beware of the units of vertical axis: it is in per 1000 not per 100 persons):

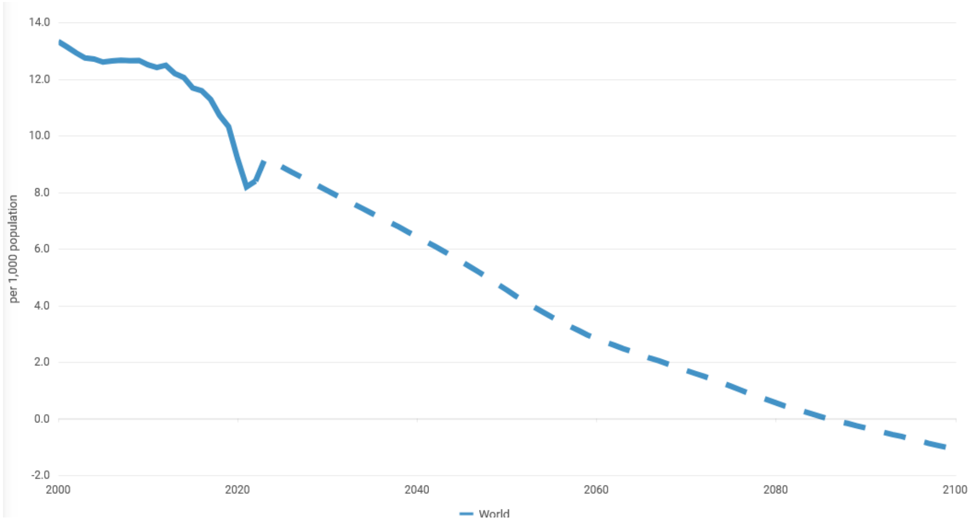

Figure C3.6: Crude rate of natural change of population

Source:
https://population.un.org/dataportal/data/indicators/53/locations/900/start/2000/end/2100/line/linetimeplot

Bottom line: the EKC does not seem an easy way out of the climate crisis. Even if this was the only driver, and even under the optimistic view, on current trends it would most probably take decades before world emissions would start to decline. This is an invitation to consider interventions on the other two components of the IPAT identity, i.e. population and affluence, as discussed in chapter 6.