Principles of sustainability economics: Extended correction guide

Chapter 1 (problems 1.1-1.8)

Problem 1.1: Principle of TBM

i. See Figure C1.1 below, which applies to problems 1.1-1.3
ii. Remembering that the formula to calculate the surface of a trapezium is the base times the average height, we obtain:

$$TB(h) = \frac{1}{2}\big(12 + MB(h)\big)h = 12h - 0.25h^2 \quad \text{[C1.1]}$$

$$\widetilde{TB}(\tilde{h}) = \frac{1}{2}\left(12 + \widetilde{MB}(\tilde{h})\right)\tilde{h} = 12\tilde{h} - 0.5\tilde{h}^2 \quad \text{[C1.2]}$$

An alternative way of finding these expressions is by computing the integral of $MB(h)$ and $\widetilde{MB}(\tilde{h})$:

$$TB(h) = \int MB(h)\,dh = \int (12 - 0.5h)\,dh = 12h - 0.25h^2$$

$$\widetilde{TB}(\tilde{h}) = \int \widetilde{MB}(\tilde{h})\,d\tilde{h} = \int (12 - \tilde{h})\,d\tilde{h} = 12\tilde{h} - 0.5\tilde{h}^2$$

Substituting $\tilde{h}$ by $(18 - h)$ in [C1.2], developing the expression of $\widetilde{TB}$ as a function of h, summing up $TB(h)$ and $\widetilde{TB}(h)$, and regrouping we finally get:

$$TB + \widetilde{TB} = 54 + 18h - 0.75h^2 \quad \text{[C1.3]}$$

Putting the derivative of this last expression equal to zero (first order condition (FOC) for maximization) we obtain the following optimal values:

$$h^* = 12, \tilde{h}^* = 6 \quad \text{[C1.4]}$$

Plugging in these values in expressions [C1.1] and [C1.2] we obtain $TB(h^*) = 108$ and $\widetilde{TB}(\tilde{h}^*) = 54$.

iii. Checking that the principle of TBM is satisfied: using [C1.4] we verify indeed that:

$$MB(h^*) = \widetilde{MB}(\tilde{h}^*) = 6 \quad \text{[C1.5]}$$

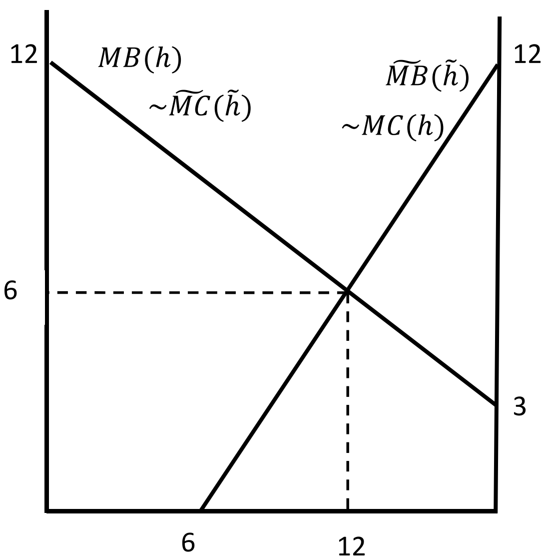


Figure C1.1: MB and MC schedules for problems 1.1-1.3

Problem 1.2: Principle of TCM

i. We first replace $\tilde{h}$ by $18-h$ in $\widetilde{MB}(\tilde{h})$ and replace h by $18-\tilde{h}$ in $MB(h)$ to find the expressions for marginal costs i.e. $MC(h) = -6 + h$ (for $h \geq 6$) and $\widetilde{MC}(\tilde{h}) = 3 + 0.5\tilde{h}$. Second, to find the total cost on leisure ($TC(h)$) and on blueberries $\widetilde{TC}(\tilde{h})$) we use the triangle and trapezium formulas and obtain:

$$TC(h) = \frac{1}{2}\big(MC(h)\big)(h-6) = \ 18 - 6h + 0.5h^2$$

$$\widetilde{TC}(\tilde{h}) = \frac{1}{2}\left(3 + \widetilde{MB}(\tilde{h})\right)\tilde{h} = 3\tilde{h} + 0.25\tilde{h}^2$$

As in problem 1,1, we can use integrals to compute $TC(h)$ and $\widetilde{TC}(\tilde{h})$. To find $TC(h)$, we have to compute the integral for $h \geq 6$ (see figure C1.1):

$$TC(h) = \int_6^h (h-6)\,dh = [0.5h^2 - 6h]_6^h = (0.5h^2 - 6h) - (18 - 36) = 18 - 6h + 0.5h^2$$

$$\widetilde{TC}(\tilde{h}) = \int_0^{\tilde{h}} (3 + 0.5\tilde{h})\,d\tilde{h} = 3\tilde{h} + 0.25\tilde{h}^2$$

We sum $TC(h)$ and $\widetilde{TC}(\tilde{h})$ and replace $\tilde{h}$ by $18-h$ and obtain:

$$TC + \widetilde{TC} = 153 - 18h + 0.75h^2$$

We apply the FOC to determine the number of hours spent on leisure that minimizes overall total costs and obtain the same optimal values as in problem 1.1 i.e.:

$$h^* = 12, \tilde{h}^* = 6$$

We plug these values into the $TC(h)$ and $\widetilde{TC}(\tilde{h})$ expressions above and obtain that $TC(h^*) = 18$ and $\widetilde{TC}(\tilde{h}^*) = 27$.

ii. The TCM principle is satisfied for $h^* = 12, \tilde{h}^* = 6$ since $\widetilde{MC}(\tilde{h}^*) = MC(h^*) = 6$.

Problem 1.3: Principle of $TNBM$

i. Using expressions from problems 1.1 and 1.2, the total net benefit on leisure $TNB(h)$ is given by:

$$TNB(h) = TB(h) - TC(h) = -18 + 18h - 0.75h^2$$

We use the FOC to determine the optimal number of hours spent on leisure and find that $h^* = 12$. We plug this value in the expression of total net benefit obtained above and find that $TNB(h^*) = 90$. We notice that this result is indeed equal to the difference between the values obtained in problems 1.1 and 1.2 i.e. $TB(h^*) = 108$ and $TC(h^*)$ =18.

ii. The principle of $TNBM$ is satisfied since $MC(h^*) = MB(h^*) = 6$

iii. Using expressions from problems 1.1 and 1.2, the total net benefit on leisure $\widetilde{TNB}(\tilde{h})$ is given by:

$$\widetilde{TNB}(\tilde{h}) = \widetilde{TB}(\tilde{h}) - \widetilde{TC}(\tilde{h}) = 9\tilde{h} - 0.75\tilde{h}^2$$

We use the FOC to determine the optimal number of hours spent on leisure and find that $\tilde{h}^* = 6$. We plug this value in the expression of total net benefit obtained above and find that $\widetilde{TNB}(\tilde{h}^*) = 27$. This result is again equal to the difference between the values obtained in problems 1.1 and 1.2 i.e. to $\widetilde{TB}(\tilde{h}^*) = 54$ and $\widetilde{TC}(\tilde{h}^*) = 27$. The principle of $TNBM$ is satisfied since $\widetilde{MC}(\tilde{h}^*) = \widetilde{MB}(\tilde{h}^*) = 6$.

Problem 1.4: Many alternatives

i. As there is a single mode of time use, namely leisure, time is used for this activity until $MB = 0$, which happens for half of the available time (i.e. $h^* = a$, see figure C1.2). This means that the remaining available time is left idle. Opportunity costs are zero ($TC(h) = 0$) so total benefit is equal to total net benefit; it is given by $TB = TNB = \frac{1}{2}a^2$.

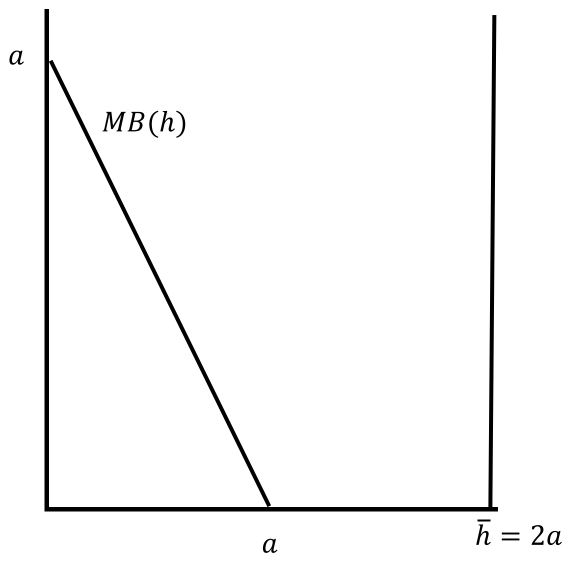


Figure C1.2: no alternative to leisure ($n = 0$)

ii. For $n = 1$, both activities can be performed until their marginal benefit is just equal to zero. As shown by figure C1.3 this happens for $h^* = \tilde{h}^* = a$. Opportunity costs are still zero for both activities so total benefit is equal to total net benefit; as in point i, it is given by $TNB(h^*) = TNB(\tilde{h}^*) = \frac{1}{2}a^2$. Therefore, $OTNB = a^2$.

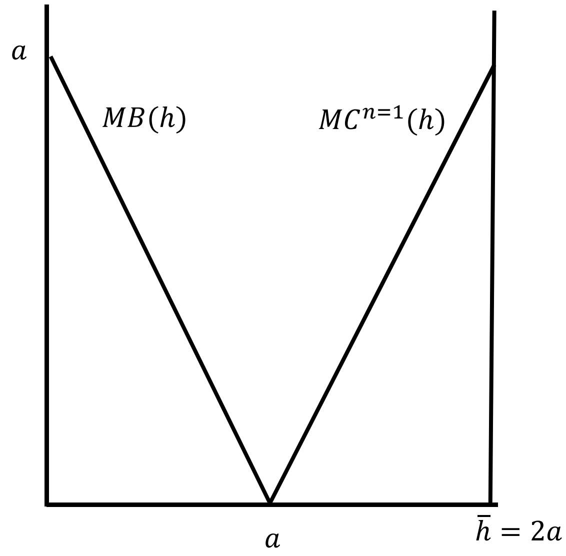


Figure C1.3: one alternative to leisure ($n = 1$)

For $n \geq 2$, the alternative activities for leisure must be combined in order to equalize their marginal benefit (see figure C1.5). For any given value of the marginal benefit, say v, this means that $\tilde{h}_1 = a - v$, and $\tilde{h}_2 = a - v$, etc. Thus, we have $\tilde{h} \equiv \sum_{i=1}^{n} \tilde{h}_i = na - nv$, which leads to $\widetilde{MB}(\tilde{h}) = a - \frac{1}{n}\tilde{h}$. Also, as we know that $h + \tilde{h} = \bar{h} = 2a$, we obtain by combining the last two expressions the marginal cost of leisure: $MC(h) = a - \frac{1}{n}(2a - h) = a\frac{n-2}{n} + \frac{1}{n}h$. The intersection between $MC(h)$ and $MB(h) = a - h$ (applying the principle of NBM) leads to: $h^* = a\frac{2}{1+n}$, $\tilde{h}^* = a\frac{2n}{1+n}$ and $MB(h^*) = \widetilde{MB}(\tilde{h}^*) = a\frac{n-1}{1+n}$. Figure C1.4 represents the particular cases for $n = 2$ and $n = 3$ (see also point iii. below).

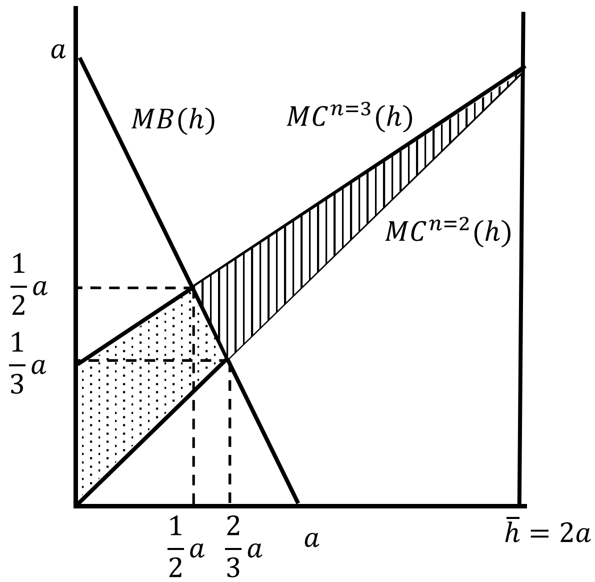


Figure C1.4: two or three alternatives to leisure ($n = 2,3$)

Using the usual formulas to calculate the corresponding areas we get the following expressions (valid for $n \geq 2$):

$$TB(h^*) = \frac{1}{2}\left[a + a\frac{n-1}{1+n}\right]h^* = a^2\frac{2n}{(1+n)^2},\ TC(h^*) = \frac{1}{2}\left[a\frac{n-2}{n} + a\frac{n-1}{1+n}\right]h^* = a^2\frac{2n^2-2n-2}{n(1+n)^2}$$

$$\widetilde{TB}(\tilde{h}^*) = \frac{1}{2}\left[a + a\frac{n-1}{1+n}\right]\tilde{h}^* = a^2\frac{2n^2}{(1+n)^2},\ \widetilde{TC}(\tilde{h}^*) = \frac{1}{2}\left[a\frac{n-1}{1+n}\right](a - h^*) = a^2\frac{(n-1)^2}{2(1+n)^2}$$

From this we obtain $TNB(h^*) = TB(h^*) - TC(h^*) = a^2\frac{2n+2}{n(1+n)^2}$, $\widetilde{TNB}(\tilde{h}^*) = \widetilde{TB}(\tilde{h}^*) - \widetilde{TC}(\tilde{h}^*) = a^2\frac{3n^2+2n-1}{2(1+n)^2}$ and $OTNB = TNB(h^*) + \widetilde{TNB}(\tilde{h}^*) = a^2\frac{3n^3+2n^2+3n+4}{2n(1+n)^2}$. This allows completing the following table:

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$MB(h^*)$	0	$\frac{1}{3}a$	$\frac{1}{2}a$	$\frac{3}{5}a$	$\frac{2}{3}a$
$OTNB$	a^2	$\frac{7}{6}a^2$	$\frac{7}{6}a^2$	$\frac{6}{5}a^2$	$\frac{37}{30}a^2$

iii. Time becomes scarce when the scarcity rent (MB) becomes positive i.e. from $n = 2$ onwards, as illustrated by Figure C1.5 below.

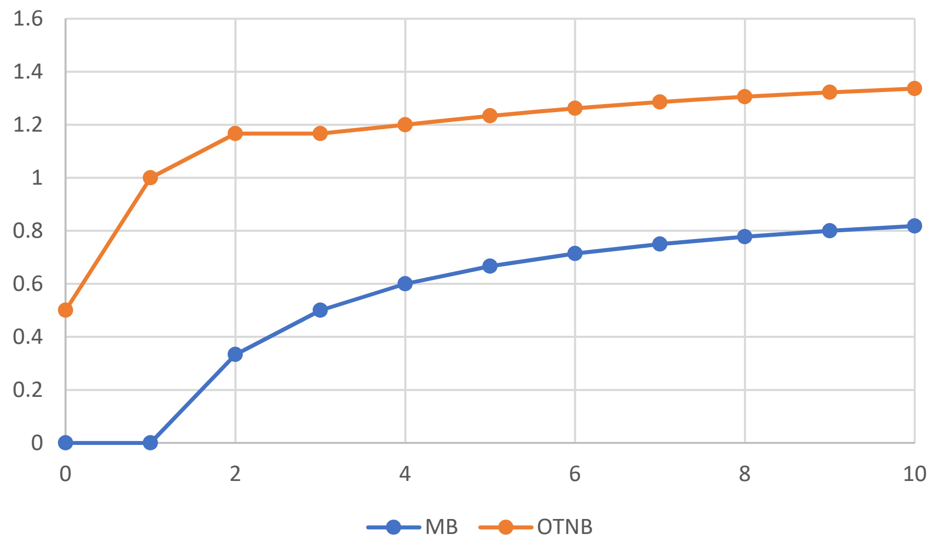


Figure C1.5: Scarcity rent (MB) and net welfare ($ONTB$) as functions of the number of alternatives (a^2 units on the vertical scale)

Regarding the "plateau" for net welfare ($ONTB$) reached between $n = 2$ and $n = 3$ it can be explained by two opposite effects when the number of alternatives increases. On the one hand, there are more sources of benefits, which leads to an increase in net benefit (from alternatives to leisure) represented by the dashed area in figure C1.4. On the other hand, there are more sources of opportunity costs, which leads to a decrease of net benefit (from leisure) represented by the dotted area in figure C1.4. As it happens, the two area perfectly compensate in this case. This result is just due to our specific setting. It can be checked on the diagram that if we had assumed a larger vertical intercept for the marginal benefit of leisure, $ONTB$ would have decreased between $n = 2$ and $n = 3$.

Problem 1.5: Trade equilibrium

i. First, we find the expression for the marginal opportunity cost of blueberries, $MC(h)$, as the marginal benefit of the alternative, $\widetilde{MB}(\tilde{h})$, where time is replaced by its complement to total time available, $\tilde{h} = 180 - h$. This leads to: $MC(h) = -360 + 4h$. Second, for each individual, we apply the principle of $TNBM$ ($MB = MC$) and obtain (see panels (a) and (b) of figure C1.6):

For Mrs. Robinson: $MB^R(h) = MC(h) \;\Rightarrow\; 360 - h = -360 + 4h \Rightarrow h^* = 144,\; MC(h^*) = 216$
For Mr. Friday: $MB^F(h) = MC(h) \;\Rightarrow\; 360 - 2h = -360 + 4h \Rightarrow h^* = 120,\; MC(h^*) = 120$

ii. The willingness to sell (for Mr. Friday) and buy (for Mrs. Robinson) are obtained by taking the relevant difference between production (read along the marginal cost curve) and consumption (read along the marginal benefit curve) for different price levels within the possible interval i.e. $120 < p < 216$. This leads to:

For Mrs. Robinson:

$WB: t^R = c^R - q^R$
$c^R: MB^R(h) = p \;\Rightarrow\; h = 360 - p$
$q^R = q^F: MC(h) = p \;\Rightarrow\; 4\tilde{h} = 360 - p \;\Rightarrow\; 4(180 - h) = 360 - p \;\Rightarrow\; h = 90 + 0.25p$

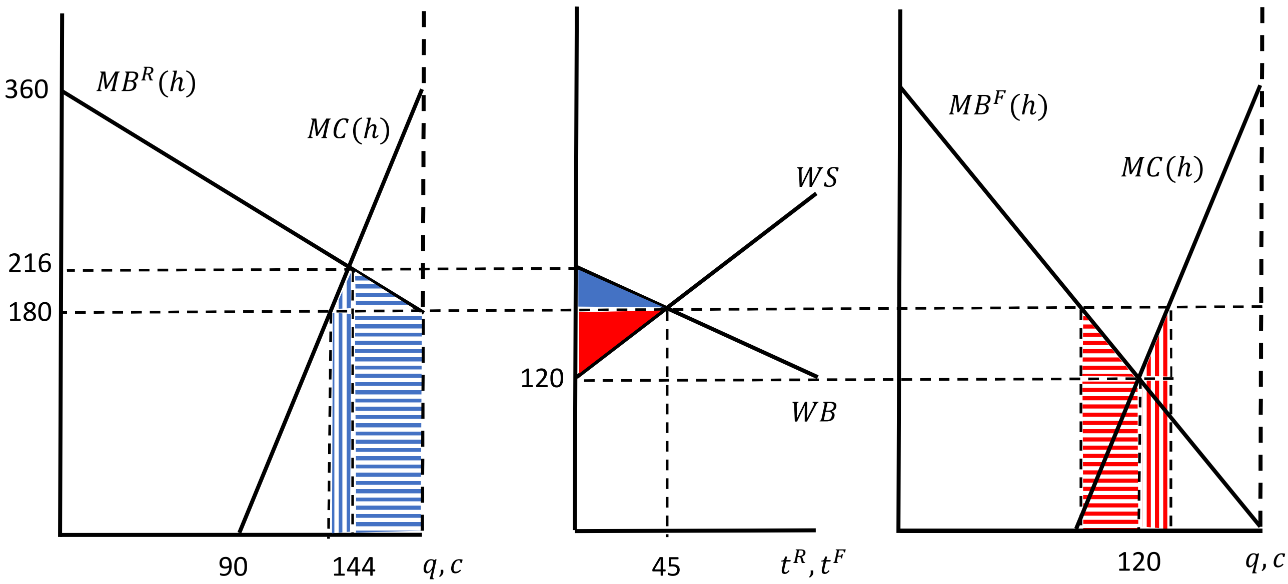
So, we obtain: $WB = (360 - p) - (90 + 0.25p) = 270 - 1.25p$

For Mr. Friday:

$WS: t^F = q^F - c^F$
$c^F: MB^F(h) = p \;\Rightarrow\; h = 180 - 0.5p$

So, we obtain: $WS = (90 + 0.25p) - (180 - 0.5p) = -90 + 0.75p$

Intersection of the two schedules leads to $p = 180$, $t^R = t^F = 45$ (see panel (b) of Figure C1.6).



(a) Mrs. Robinson (b) trade (c) Mr. Friday

Figure C1.6: trade equilibrium

By calculating the corresponding areas below the marginal benefit and marginal cost schedules we can calculate the welfare changes generated by trade for the two agents:

For Mrs. Robinson (panel (a) of figure C1.6):

Increase in TB (blue horizontally hatched area): $\frac{1}{2}(216 + 180)36 = 7'128$
Decrease in TC (blue vertically hatched area): $\frac{1}{2}(216 + 180)9 = 1'782$
Gains from trade (considering her payments): $(7'182 + 1'789) - (180 * 45) = 810$

For Mr. Friday (panel (c) of figure C1.6):

Increase in TC (red vertically hatched area): $-\frac{1}{2}(120 + 180)15 = 2'250$
Decrease in TB (red horizontally hatched area): $-\frac{1}{2}(120 + 180)30 = 4'500$
Gains from trade (considering his earnings): $(180 * 45) - (2'250 + 4'500) = 1'350$

Total gains from trade: $810 + 1'350 = 2'160$.

We can verify these results by calculating the gains from trade as the triangular areas in panel (b) of figure C1.6:

For Mrs. Robinson (blue area): $\frac{1}{2}(216 - 180)45 = 810$

For Mr. Friday (red area): $\frac{1}{2}(180 - 120)45 = 1'350$.

Problem 1.6: Negative externality

i. To find the market equilibrium (point B in figure C1.7) we solve the $MB = MC_p$ equation: $12 - x = 3 + 0.5x$, which leads to $x = c = q = 6, p = 6$. From this we can calculate the net social benefits for the upper part (area ABC) and bottom part (area $DEBC$) of the village:

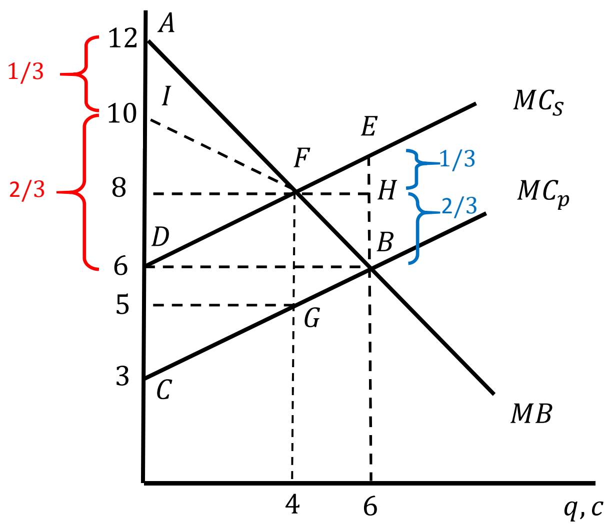


Figure C1.7: Negative externality

$$TNB_{up} = \frac{1}{2} \cdot 6 \cdot (12 - 3) = 27,\ TNB_{producers}: \frac{3 \cdot 6}{2} = 9\ and\ TNB_{consumers}: \frac{6 \cdot 6}{2} = 18$$
$$TNB_{down} = -3 \cdot 6 = -18$$

Which is quite an unequal distribution of the total net social benefit which is given by $TNB_{tot} = TNB_{up} + TNB_{down} = 9$

ii. To find the social optimum (point F) we solve the $MB = MC_s$ equation: $12 - x = 6 + 0.5x$, which leads to $x = c = q = 4, p = 8$. The net social benefits for the upper part (area $AFGC$) and bottom part (area $DFGC$) of the village are given by:

$$TNB_{up} = 4 \cdot \frac{(12 - 3) + (8 - 5)}{2} = 24,\ TNB_{prod}: 4 \cdot \frac{5 + 3}{2} = 16\ and\ TNB_{cons}: \frac{4 \cdot 4}{2} = 8$$
$$TNB_{down} = -3 \cdot 4 = -12$$

The total net social benefit is larger, at $TNB_{tot} = TNB_{up} + TNB_{down} = 12$, i.e. an increase of 3 from point i, but its distribution is still quite unequal.

iii. If the property rights are given to the upper part of the village, the victims can offer up the 3 as a compensation for each unit of reduction. The upper part will accept until its marginal loss of reduction becomes larger than 3 that is when $q = 4$, which is optimal. The upper part of the village gets $AFGC$ plus full compensation (FBG) plus 2/3 of the net gain (FEB). The bottom part suffers a loss of $DFGC$ and has to transfer $FBG + FHB$ to the upper part. Thus:

$$TNB_{up} = 24 + \frac{1}{2} \cdot 3 \cdot 2 + \frac{2}{3} \cdot \frac{1}{2} \cdot 3 \cdot 2 = 29$$
$$TNB_{down} = -12 - \frac{1}{2} \cdot 3 \cdot 2 - \frac{2}{3} \cdot \frac{1}{2} \cdot 3 \cdot 2 = -17$$

This is very unequal but net total welfare is maximized, with $TNB_{tot} = TNB_{up} + TNB_{down} = 12$.

If the property rights are attributed to the bottom part of the village, the victims ask for a minimum compensation of 3 per additional unit. This is OK for the upper part as long as its marginal gain is larger than 3 that is when $q = 4$, which is optimal. The upper part of the village gets area $AFGC$ minus a transfer corresponding to the full compensation of the victims ($DFGC$) and 2/3 of the net social gain (AFD). The bottom part suffers no loss (they are fully compensated) and still gets 2/3 of the net social gain (AFD). This leads to:

$$TNB_{up} = 24 - 12 - \frac{2}{3} \cdot \frac{1}{2} \cdot 6 \cdot 4 = 4$$
$$TNB_{down} = \frac{2}{3} \cdot \frac{1}{2} \cdot 6 \cdot 4 = 8$$

Net total welfare is maximized again, with $TNB_{tot} = TNB_{up} + TNB_{down} = 12$, but this time the allocation is less unequal (and privileges the victims).

Problem 1.7: Intergenerational linkages

i. We equalize the marginal benefit (MB) with the marginal extraction cost of the depletable resource (MEC_L) for the first generation: $12 - 0.5q = 4$, which leads to $q^* = 16$. What is left for generation 2 is 8 units of the depletable resource (24-16), which corresponds to a marginal benefit of $12 - 0.5(8) = 8$, which is smaller than MEC_S, so there is no incentive to consume the renewable resource at a higher marginal cost. The last marginal unit extracted in period 1 would have generated a net benefit of 8-4=4 if it had been extracted in period 2, so the MUC of generation 1 is equal to 4. In figure C1.8, the net social welfare is given by area ABC for generation 1 and area $DEFB$ for generation 2, that is:

$$TNB_1 = \frac{1}{2} \cdot 8 \cdot 16 = 64$$
$$TNB_2 = 8 \cdot \frac{8+4}{2} = 48$$

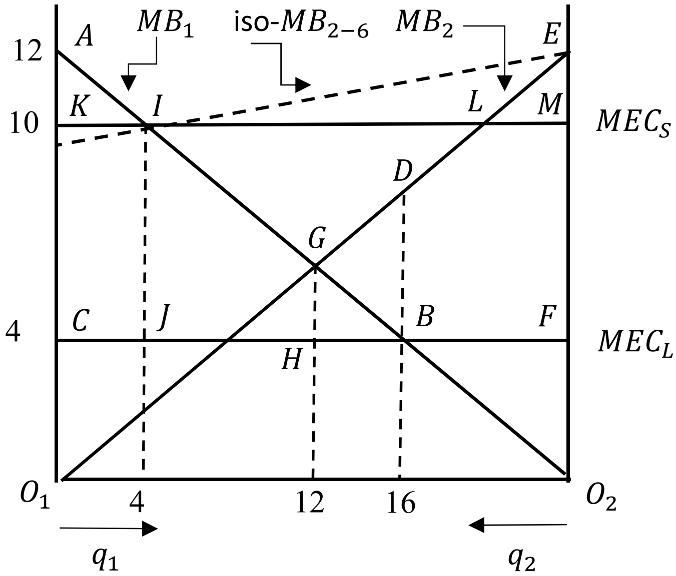


Figure C1.8: Efficient and inefficient extraction

ii. In this case the available quantity of the depletable is split equally between the two generations (point G of figure C1.8) i.e. $q^* = 12.$, which corresponds to a marginal benefit of $12 - 0.5(12) = 6$. The last marginal unit extracted in period 1 would have generated a net benefit of 6-4=2 if it had been extracted in period 2, so the MUC of generation 1 is equal to 2. The net social welfare is given by area $AGHC$ for generation 1 and area $GEFH$ for generation 2, which leads to:

$$TNB_1 = 12 \cdot \frac{8+2}{2} = 60$$
$$TNB_2 = TNB_1 = 60$$

iii. For a given number of generations, optimal exploitation implies the equality between marginal benefit and total marginal cost (see principle 1.7). The number of generations is itself optimized when total marginal cost (equal to marginal benefit) is equal to the marginal extraction cost of the renewable resource, $MEC_S = 10$ in our case (an alternative way to find the optimal number of generations would have been to add additional generations one by one until the iso-MB line of future generations intersects the MB line of generation 1 rightly at or slightly above the MEC_S level, so that the total scarcity rent is fully exploited, as in figure 1.15). Equalizing marginal benefit to total marginal cost leads to: $12 - 0.5q = 10$, i.e. $q^* = 4$ for each generation. As the total available quantity of the depletable is 24 units, this means that the first six generations can rely on the depletable, with a net social gain given by area $AIJC$, while subsequent generations exploit only the renewable, with a net social gain given by area AIK. That is:

$$TNB_1 = TNB_2 = \cdots = TNB_6 = 4 \cdot \frac{8+6}{2} = 28$$

$$TNB_{i>6} = \frac{1}{2} \cdot 2 \cdot 4 = 4$$

Considering that at point i. all subsequent generations have to rely on the renewable, that corresponds to a change of -36 for generation 1, -20 for generation 2 and +24 for generations 3,4,5 and 6 (with no changes for further generations). In other words, switching from the fully inefficient (point i.) to the fully efficient extraction trajectory (point iii.) leads to an increase in overall net welfare of +40, which corresponds to area $ILDB$ in figure C1.8 (full exploitation of the scarcity rent).

Problem 1.8: Common pool resources

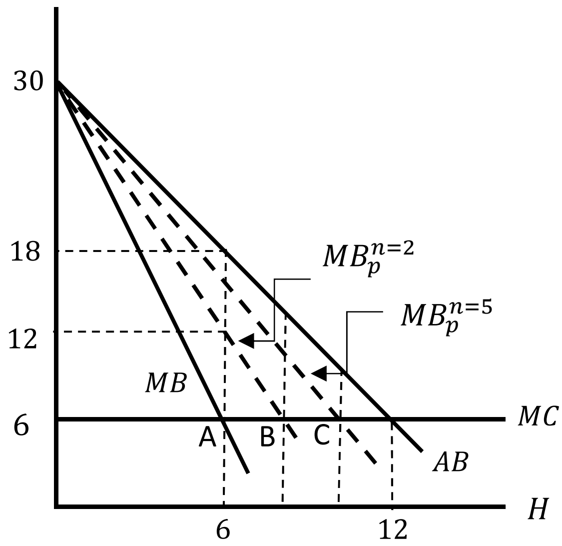


Figure C1.9: Optimal exploitation and overexploitation of a fish stock

i. To determine an expression for the marginal benefit we first need to calculate the expression for total benefit, $TB = H \cdot AB = 30H - 2H^2$, and then takes its derivative which leads to $MB = 30 - 4H$

To determine the *socially* efficient total effort level (H^*) we find the coordinates of point A, where MB is equal to MC: $30 - 4H = 6$, which leads to $H^* = 6, p^*(AB^*) = 18$

ii. The perceived marginal benefit of an individual agent (with effort level h) is AB minus the loss on all infra-marginal units i.e.:

$$MB_p^n = AB + h\frac{\partial AB}{\partial H}\frac{\partial H}{\partial h}$$

where $\frac{\partial AB}{\partial H} = -2$ and $\frac{\partial H}{\partial h} = 1$ to account for the fact that each agent is selfishly assuming that she is the only one to deviate from the optimal rule where $H^* = nh^*$.

For $n = 2$, we have therefore $MB_p^{n=2} = (30 - 2 \cdot 2h) - 2h = 30 - 6h$, and as all agents behave the same way we can replace h by $H/2$ to obtain $MB_p^{n=2} = 30 - 3H$. The intersection with the MC (point B) leads to $H = 8$ and $AB = 14$, so the degree of over-exploitation is 33.33% (i.e. (8-6)/6).

iii. Following the same logic:

For $n = 5$, we have $MB_p^{n=5} = (30 - 2 \cdot 5h) - 2h = 30 - 12h$, and as all agents behave the same way we can replace h by $H/5$ to obtain $MB_p^{n=5} = 30 - \frac{12}{5}H$. The intersection with the MC (point C) leads to $H = 10$ and $AB = 10$, so the degree of over-exploitation is 66.66% (i.e. (10-6)/6). The larger the number of agents, the larger the incentive to free-ride and the larger the over-exploitation of the resource.